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A Plain and Comprehenſive

TREATISE
OF
Decimals and Menſuration.

Calculated for the
INSTRUCTION and IMPROVEMENT of thoſe who are deſirous to be acquainted with FIGURES, in accurate and uſeful COMPUTATIONS.

CONTAINING,

I. AN Application of Decimals to all the leading Branches of ARITHMETIC, in a Variety of Examples; together with the Doctrine of Circulating Decimals or Repetends.

II. INTEREST in all its Caſes, both Simple and Compound; with Conſtructions upon the Rules of Rebate or Diſcount: Likewiſe ſeveral new Methods adapted to the Valuing of Leaſes or Annuities on Lives.

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The Whole rendered familiar to the meaneſt Capacity.

By WM. CRUMPTON.

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Hist of Sci
1-15-27

The Editor *to the* Public.

AMONG the many improvements of late years that have been made in arithmetic, the invention of DECIMALS is the moſt prevalent of any, for its diſpatch in numbers, and accuracy in computation. Its utility in matters of buſineſs and concern has induced me to put forth a Tract to endeavour to render DECIMALS more familiar than has hitherto been; including ſeveral uſeful matters that are new and conciſe, and which are laid down in the plaineſt manner, with the work to each example.

IN matters of this kind an Editor can't be too careful in the circumſpection of every article he inſerts; as alſo in reviſing the ſame; yet after all, there are very few performances that can pleaſe every taſte; for ſome are too apt to cavil if they do but meet with things out of their own way, while others are as much remiſs in furniſhing the mind with any kind of improvement whatever: But learning creates a beneficial notion of the moſt intereſting concerns of life, and forms a man for ſociety.

Mr

My deſign in introducing Practical Muſic, is conſidered with this view, that it may be a means of relaxation at any time when the mind is rather wearied with too cloſe an application to ſtudy or buſineſs.

But the chief inducement of the whole undertaking has been more for the public good than any lucrative views of my own; therefore if any ſubject matter ſhould appear not couched in the moſt engaging terms, it is hoped all cenſure on that head may be ſilent, as this piece was wholly deſigned for general uſe.

The Subſcription I have met with in my undertaking, together with the ſanction of ſeveral learned and ingenious Gentlemen, are motives which have far exceeded my moſt ſanguine hopes: Therefore 'tis my ardent wiſh, that the learner may meet with the deſired ſatisfaction in the following pages.

A PLAIN and COMPREHENSIVE
TREATISE
OF
DECIMALS
AND
MENSURATION.

Decimal Arithmetic.

DECIMALS were invented for the eaſy working of fractional parts, by a much more commodious and compendious method than that of Vulgar Fractions, and is therefore now become of general uſe in all the branches of arithmetic; for the ſeveral operations perform'd in decimals, differ but little from thoſe of whole numbers.

A fraction whether vulgar or decimal is ſome part or parts of a unit. To expreſs it as a vulgar fraction it is written down as it comes from diviſion; thus, if 37 was to be divided by 5, the quotient wou'd be more than 7, and leſs than 8; that is, it would be $7\frac{2}{5}$, or 7 times and 2 parts of 5: Here, the uppermoſt number of the fraction is call'd the Numerator, and that under the line is the Denominator or whole number.

Now decimal fractions are wrote without their denominators, for the integer or whole thing being always underſtood to conſiſt of an unit with as many cyphers annex'd to the right hand, as the decimal fraction has places; and here the numerator or decimal itſelf is diſtinguiſh'd from whole numbers by a ſeparating point; thus, 5.4 is $5\frac{4}{10}$; and 0.7 is $\frac{7}{10}$; 35.05 is $35\frac{5}{100}$, &c. But the different value of the ſeveral places will more plainly appear from the following Table.

Whole numbers.							Decimal parts.						
	6	5	4	3	2	1	.1	2	3	4	5	6	
&c.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units place.	Parts of Ten, or $\frac{1}{10}$.	Parts of a Hundred.	Parts of a Thouſand.	Parts of Ten Thouſand.	Parts of a 100 Thouſand.	Parts of a Million.	&c.

From whence it is evident, that as whole numbers *increaſe* by a tenfold proportion towards the left hand; ſo decimal parts *decreaſe* in the ſame proportion towards the right.

EXAMPLE. Write down or expreſs, fifteen parts of a hundred, in decimals.

Anſwer .15

Again, Write down fifty two integers, and ſeventy five parts of one thouſand the integer.

Anſwer 52.075

N.B. There are finite decimals, and thoſe which are infinite. A finite decimal is that which ends at a certain number of places; but an infinite, that which no where ends, and theſe are called circulating decimals or repetends, and diſtinguiſhed by a daſh upon the figure, and are ſuch wherein one or more figures are continually repeated; as if 30 was to be divided by 9, the quotient would be $3.\acute{3}$; the decimal being a continual circulation of 3's, equal to the fraction of $\frac{3}{9}$. Thus $24.3\acute{6}66$, &c. is call'd a ſingle circulate, or repetend of 6's; and $6.\acute{2}6\acute{4}$, &c. is call'd a compound circulating decimal, and has the firſt and laſt of the recurring digits daſhed accordingly. Otherwiſe any of theſe expreſſions may be continued at pleaſure, by repeating the circulating figure or figures without the daſh.

In all operations, if the reſult conſiſts of ſeveral nines in the decimal place, reject them, and make the next ſuperior place an unit more; thus for 6.23999, &c. write 6.24.

ADDITION *and* SUBTRACTION,

CASE I.

To add or ſubtract finite decimals.

RULE.

HAVING ſet down all the propos'd numbers in their reſpective places (as in addition, &c. of whole numbers) viz. every figure as well of the decimal parts as of the whole number directly underneatth thoſe of the ſame value or name, which may be very eaſily done if the ſeparating points are placed directly under one another. Then, add or ſubſtract them as if they were all whole numbers, and for their ſum or difference, cut off by the ſeparating point, ſo many places of decimal parts as there are in any of the given numbers.

EXAMPLES *in* ADDITION.

102.73	19.3745	8219.801
45.0264	5.02	0.54
3.141	196.7	59.32783
563.0	3278.92303	.456
8.535	36.14	424.535207
722.4324	3536.15753	8704.660037

EXAMPLES

EXAMPLES in SUBTRACTION.

From	192.6521364	7163760.15
Take	78.461538	3901842.08924
	114.1905984	3261918.06076

CASE II.

To add or ſubtract decimals with ſingle circulates or repetends.

RULE.

Make every line to end at the ſame place by filling up the vacancies with the recurring digits, and to the finite terms you may annex a cypher or cyphers, then add as before; only increaſe the ſum of the right hand row with as many units as it contains nines, and the figure in the ſum under that place will be a circulate.

And for ſubtraction, fill up and continue as in the following examples; and it the right hand of the ſubtrahend be greater than the upper figure in the minuend, inſtead of borrowing 10 as uſual, borrow 9 in this place, the reſt as uſual, and the right hand figure of the remainder will be a circulate.

EXAMPLES.

Add

$471.641\dot{3}$
15.2
3217.846
$3.12\dot{7}$

Subtract.

$8.147\dot{6}$ $412.3\dot{8}$
3.974 $16.649\dot{7}$

In theſe examples you obſerve there are ſingle recurring figures, which before you add or ſubtract, muſt be made to end together.

	See the ſame filled up.	
$471.641\dot{3}$		
15.2000		
3217.8460	$8.147\dot{6}$	$412.3\dot{2}22$
$3.12\dot{7}7$	3.9740	$16.649\dot{7}$
$3707.815\dot{1}$	$4.173\dot{6}$	$395.672\dot{4}$

24.7	$7.64\dot{3}33$
$12.\dot{2}2$	2.7646
$12.4\dot{7}$	$4.8787\dot{3}$

Here it may be obſerved, that in each Example the circulates are carried one place farther than the finite expreſſions.

MULTIPLICATION.

CASE I.

When both the factors are finite decimals.

RULE.

WHETHER the factors, viz. the number propos'd to be multiplied together, are either all decimal parts, or decimals join'd to whole numbers, multiply them together as if they were all whole numbers; only obſerve to point off ſo many decimal places on the right hand of the product as are in both the multiplicand and multiplier.

But when it ſo happens that there are not ſo many figures in the product as there are decimal places requir'd, in that caſe you muſt ſupply the defect by prefixing cyphers to the left hand of the product.

EXAMPLES.

Multiply .45 into itſelf: And 1.7345 by .69.

```
   .45             .69
  ----           ------
   225           156105
  180           104070
  ----          --------
 .2025          1.196805
 -----          --------
```

Multiply 560.138 by 3.087:

```
      3.087
     ------
    3920966
   4481104
 1680414
 -----------
 1729.146006
 -----------
```

And .01472 by .1045.

```
      .1045
     ------
       7360
      5888
    1472
 ----------
 .001538240
 ----------
```

CASE II.

If the right hand figure of the multiplicand is a circulate.

RULE.

Multiply the multiplicand as before, by every figure in the multiplier; obſerving to increaſe the right hand figure of each reſulting line, by as many units as there are nines in the firſt product of that line; and the right hand figure of each line will be a circulate; therefore in adding them together, make them end at the ſame place, as ſhewn in addition.

EXAMPLES.

$$\begin{array}{r}
6174.4\dot{3} \\
86.2 \\
\hline
123488\dot{6} \\
3704660\dot{0}0 \\
4939546\dot{6}66 \\
\hline
532236.15\dot{3} \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
5167.924\dot{3} \\
156 \\
\hline
31007546\dot{0} \\
25839621\dot{6}6 \\
51679243\dot{3}33 \\
\hline
806196.196\dot{0} \\
\hline
\end{array}$$

CASE III.

If the multiplier has a ſingle repetend.

RULE.

Multiply by it as though it was a finite digit, ſetting the product of this firſt line one place forwarder than ordinary, towards the left hand; divide the reſult by nine, continuing the quotient (if needful) till it arrives at a circulate; the reſt of the work as uſual; then beginning at the place under the right hand figure of the multiplicand or multiplier, cut off for fractions, and this will be the true product.

EXAMPLES.

518.275	4260.95217
.646	1.43
9)3109650	9)1278285651
3455166	1420317390
2073100	1704380868
3109650	426095217
335.151166	6107.3647770

CASE IV.

When both multiplicand and multiplier have each a ſingle circulate.

RULE.

In multiplying the multiplicand by each figure in the multiplier, obſerve Caſe II. but the firſt line (or that line produced by multiplying the multiplicand by the repetend in the multiplier) muſt be order'd by Caſe III.

EXAMPLES.

EXAMPLES.

$$\begin{array}{rr}
62.512\dot{8} & \\
542.\dot{4} & \\
\hline
9)250051\dot{5} & \\
\hline
27783\dot{5}061, \&c. & \\
125025\dot{7}77, \&c. & \\
250051\dot{5}555, \&c. & \\
312564\dot{4}4444, \&c. & \\
\hline
33909.769\dot{2}83, \&c. & \\
\hline
\end{array}
\qquad
\begin{array}{rl}
4\dot{8}.75\dot{4} & \\
72.1\dot{3} & \\
\hline
9)14626\dot{3} & \\
\hline
1625\dot{1}4\dot{8} & 1 \\
4875\dot{4}44 & 4 \\
9750\dot{8}888 & 8 \\
34128\dot{1}1111 & 1 \\
\hline
3516.820\dot{5}9\dot{8} & \\
\hline
\end{array}$$

Note. From the laſt example it is eaſy to conceive how compound circulates are generated; and if it was carried on continually, it wou'd ſtill repeat the ſame.

EXAMPLES, having a compound repetend in the multiplicand.

$$\begin{array}{rl}
6.84\dot{3}2\dot{6} & 2 \\
4.7 & \\
\hline
47902\dot{6}\dot{4} & 2 \\
27373\dot{0}\dot{5}3 & 0 \\
\hline
32.\dot{1}63\dot{3}1\dot{7} & \\
\hline
\end{array}
\qquad
\begin{array}{rl}
54\dot{7}.462\dot{4} & 7 \\
32 & \\
\hline
109\dot{4}924\dot{9} & 4 \\
164\dot{2}387\dot{4}2 & 2 \\
\hline
17518.799\dot{1} & \\
\hline
\end{array}$$

Here you multiply as uſual, adding ſo many units to the right hand place of the product made by each digit, as there are tens in the product made by the ſaid digit, and the left hand leading digit of the repetend; each of which products contains a repetend equal in number of places to that in the multiplicand, which being made conterminous, and added together, gives a repetend alſo, of the like number of places in the general product.

EXAMPLE of a compound repetend in the multiplier.

Let 23.164 be multiplied by $4.\dot{2}\dot{5}$.

$$\begin{array}{r} 23.164 \\ .\dot{2}\dot{5} \\ \hline 115820 \\ 46328 \\ \hline \end{array}$$

$.99)5.79100(5.8\dot{4}\dot{9}$ product of repetends.

$$\begin{array}{l} 495 \\ \hline \;\;841 \\ \;\;792 \\ \hline \;\;\;\;490 \\ \;\;\;\;396 \\ \hline \;\;\;\;\;\;940 \\ \;\;\;\;\;\;891 \\ \hline \;\;\;\;\;\;\;\underbrace{49} \end{array}$$

$$\begin{array}{r} 23.164 \\ 4 \\ \hline 92.656 \\ 5.8\dot{4}\dot{9}49 \\ \hline \text{True product } 98.505\dot{4}\dot{9} \\ \hline \end{array}$$

Thus after multiplying by each digit of repetends as in the common way, you divide that product by as many 9's as there are repetends in the multiplier, for the product of repetends, which added to the terminate product, gives the true product. Thus you ſee the 9's give a denominator; and the value of the repetend is equal to this fraction $\frac{25}{99}$.

An EXAMPLE having a compound repetend both in the multiplicand and multiplier, viz. 111.94 multiply'd by 4.635.

```
   111.94                      111.94
    4.600                        .035
 --------                    --------
   67163                        55970
  447767                       335825
 --------                    --------
 514.93893               .99)3.91796(3.9575
   3.9575                     297
 --------                    -----
 518.89043, &c.                947
   Product.                    891
                               -----
                                569
                                495
                                -----
                                 746
                                 693
                                 -----
                                  537
                                  495
                                  -----
                                   42
```

N.B. When any number is to be multiplied by a repetend, in ſuch caſe, multiply by the repetend as if it was a terminate number, and divide by the denominator of the repetend, viz. 9, or 99, or 999, or 9999, &c.

DIVISION.

DIVISION.

CASE I.

When the diviſor and dividend are finite decimals.

DIVIDE as in diviſion of whole numbers; and to value the quotient, obſerve either of the following rules.

RULE I.

The firſt figure in the quotient muſt be of the ſame denomination with that figure in the dividend which ſtands (or is ſuppos'd to ſtand) over the units place in the diviſor, at firſt ſeeking.

II. The quotient muſt always have ſo many decimal places, as the dividend has more than the diviſor.

Note 1ſt. If the diviſor and dividend have both the ſame number of decimal places, the quotient will be a whole number.

2d. If the dividend hath not ſo many places of decimals as the diviſor, there muſt be a competent number of cyphers annex'd to the dividend to make it conſiſt of as many (at leaſt) or more places of decimals than the diviſor; for the cyphers added muſt be reckon'd as decimals.

3d. But if when the diviſion is ended, the quotient has not ſo many places of figures as it ſhou'd have places of decimals, then that deficiency muſt be ſupply'd with cyphers prefixt to the left hand of the quotient, with a point before them.

4th. In dividing of whole or mixt numbers, if there be a remainder, you may bring down more cyphers, and by continuing your diviſion, carry the quotient to as many places of decimals as you pleaſe.

EXAMPLES.

Let 8.142 be divided by 62: And 62 by .742.

```
62)8.142(.131      .742)62.0000(83.5
   62                   5936
   ---                  ----
   194                  2640
   186                  2226
   ---                  ----
    82                   4140
    62                   3710
    --                   ----
    20                    430
```

Divide 746.54 by 3.07: And .072 by 9.05.

```
3.07)746.54(243    9.05).0720000(.00795
     614                  6335
     ----                 ----
     1325                 8650
     1228                 8145
     ----                 ----
      974                  5050
      921                  4525
      ---                  ----
       53                   525
```

CASE

CASE II.

If there be a circulate in the dividend.

RULE.

Bring down the ſaid repetend as often as you think proper, or till you come to a repetend in the quotient.

EXAMPLES.

8)84.3$\dot{6}$(10.5458$\dot{3}$
8
—
43
40
—
3$\dot{6}$
32
—
4$\dot{6}$
40
—
6$\dot{6}$
64
—
2$\dot{6}$ remain.
24
—
2$\dot{6}$ ſame remainder.

12)42.$\dot{3}$4$\dot{4}$(3.5$\dot{2}\dot{8}$6953
36
—
6$\dot{3}$
60
—
34
24
—
104
96
—
8$\dot{3}$
72
—
114
108
—
64
60
—
4$\dot{3}$
36
—
7

N.B. If before the quotient repeats it ſhou'd require a long or tedious operation, you may leave off at what number of places may be found proper or convenient.

CASE III.

When the diviſor has a repetend.

RULE.

When any number is to be divided by a repetend, ſet the whole dividend under itſelf in ſuch manner, that each figure may be remov'd ſo many places to the right hand as are the number of circulating places in the diviſor; and in like manner remove the diviſor, but never carry the figures in the lowermoſt line any further to the right hand, than the uppermoſt line of the diviſor; then ſubtract as in whole numbers, and let the remainders be a new diviſor and new dividend; and if there be any circulating figures in the firſt dividend, regard muſt be had to the continuance of them in ſubtracting one from the other, and the like number of repetends will be in this new dividend.

EXAMPLES.

```
5.3)74
 5 74
48)66.6(13.875
    48
    186
    144
     420
     384
      360
      336
       240
       240
```

```
6.4)6.4
      64
6.4)6.336(.99
      576
       576
       576
```

N.B. You find it ſometimes happens that the repetend begins in the whole number. And it is plain to any one's obſervation, that any number of digits being divided by the ſame number of 9's will continually circulate by the figures of the dividend, and the remainder will be the ſame as the ſaid figures in the dividend. Therefore if 64 is multiplied by 100, it will exceed 64 multiplied by 99 by 64 units; the difference between 100 and 99 being unity. From whence it may be eaſily apprehended, that whenever any decimal fraction circulates or repeats, ſo many 9's will be the denominator as there are repeating places of figures in the fraction; viz. if there is a ſingle circulate 9 is its denominator, if double 99 is its denominator, &c. for $\frac{64}{99}$ is equal to $\dot{6}\dot{4}$; and $4.\dot{5}$ is $4\frac{5}{9}$. Hence appears the reaſon of ſo many units being added in addition, as are numbers of 9's in the laſt row to the right hand, and of borrowing a 9 in ſubtraction, becauſe 9 is always a denominator to theſe ſingle repetends. In multiplication alſo, we have a reaſon for that method of dividing the product by 9 when the multiplier is a ſingle repetend; or by 99 if it is a double repetend; or by 999 if it is a triple repetend, &c.

More EXAMPLES for practice.

41.7)274.693
41 274693

37.6)247.2240(6.575
2256

2162
1880

2824
2632

1920
1880

40

8.074)12.02472
807 1202472

7.267)10.822248(1.489
7267

35552
29068

64844
58136

67088
65403

1685

72.13)3516.820592
721 3516820592

64.92)3165.1385338(48.754
25968

56833
51936

48978
45444

35345
32460

28853
25968

28853

See the proof of this in Multiplication, Case IV.

The following is an anſwer to a queſtion in the *Gentleman's Magazine* for Feb. 1750, which I believe was never anſwered before.

QUESTION.

Required the true quotient of the following diviſion, with 31 figures only, as perfect as if infinitely continued by the common methods, and with infinitely leſs burden to the memory?

999999)719560075657957917.(

See the work.

719560075657957917.0000000000000
7195600756579579170000000
7195600756579579170
7195600756579
7195600
7

719560795218.7531357531357531357
Anſwer, the true quotient.

N. B. This ſame method will anſwer all the purpoſes of dividing any number by any quantity of 9's, by placing the dividend under itſelf in ſuch a repeated order as are 9's in the diviſor; and which method will be found uſeful in repetends.

REDUCTION.

REDUCTION.

CASE I.

To reduce a vulgar fraction to a decimal.

Rule.

IF to the numerator given you annex a competent number of cyphers, and divide the reſult by the denominator, the quotient is the decimal equivalent to the vulgar fraction given.

Examples.

Reduce $\frac{1}{4}$ to a decimal: And $\frac{3}{4}$ to a decimal.

4)1.00	4)3.00
.25	.75

Find the decimal equal to $\frac{9}{12}$:

12)9.00

.75

And the decimal to $\frac{10}{12}$.

12)10.00

.83

Reduce $\frac{3}{573}$ to a decimal: And $\frac{1286}{1378}$ to a decimal.

```
573)3.000000(.005235   1378)1286.0000(.9332
    2865                    12402
    ----                    -----
     1350                    4580
     1146                    4134
     ----                    ----
      2040                   4460
      1719                   4134
      ----                   ----
       3210                   3260
       2865                   2756
       ----                   ----
        345                    504
```

Note. If you ſhou'd have a compound vulgar fraction, and wou'd reduce it into a decimal, you muſt firſt reduce the compound fraction to a ſingle one, thus; multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, and proceed as before.

Reduce this compound vulgar fraction, viz. $\frac{3}{5}$ of $\frac{2}{4}$ of $\frac{1}{2}$ of any thing, to a decimal fraction.

```
        5
        4
       --
       20   3
        2   2
       --  --
       40 ) 6.00(.15
            40
            ---
            200
            200
            ---
```

And thus the compound vulgar fraction of 1d. is $\frac{1}{12}$ of $\frac{1}{20}$, equal to $\frac{1}{240}$ of a £.

CASE II.

To find the value of a decimal in the known parts of money, weights, meaſures, &c.

Rule.

Multiply the given decimal by the number of parts in the next inferior denomination, and from the product point off ſo many places to the right hand as there are places in the decimal given; and multiply thoſe figures pointed off by the number of parts in the next inferior denomination, and prick off ſo many places as before, and ſo continue to do, till you have brought it to the loweſt denomination requir'd.

Examples.

What's the value of .7566 of a pound ſterling?

```
          20
       ------
  s.   15.1320
          12
       ------
  d.    1.5840
           4
       ------                      s.  d.
  q.    2.3360  Anſwer 15 : 1½.336
       ------
```

Note, There is a more compendious method of valuing the decimal of a pound very near, only by the inſpection of the three firſt figures. Double the firſt decimal place towards the left hand (or place of primes) and if the ſecond figure be under 5, then the firſt figure only doubled will be ſhillings; but if the ſecond figure be 5, or above 5, then you muſt add one ſhilling more to thoſe you doubled; and what remains from the ſecond figure above 5, carry to the next figure, and reckon them as ſo many farthings; and if they be 14 or above, abate 1; and if 28 or above, abate or caſt away two farthings of the number.

What is the value of .712 of a foot? And the value of .3705 of a yard?

	.712		.3705	
	12		3	
In.	8.544		1.1115	
	4		12	
qu.	2.176	In.	1.3380	F. I.
		Anſ. 8¼ .176		Anſ. 1 1 1,&c.

N.B. It may not be improper in this place to ſhew how to value a vulgar fraction, as it may ſometimes be found more ready than reducing them into decimals.

RULE.

Multiply the integer (or quantity) into the numerator, and divide by the denominator; and if any thing remains, multiply it by the number contained in the next inferior denomination with reſpect to one of the integer, and proceed to divide as before.

What's the value of $\frac{5}{8}$ of a pound ſterling?

```
       s.
      20
       5
    ——————  s.  d.
  8)100(12 : 6 Facit.
     96
    ——————
      4
     12
    ——————
   8)48(6
     48
     ——
```

What value is 240 times $\frac{4}{15}$ of a penny?

```
         4
      ——————
    ⎧ 3)960
 15 ⎨ ——————
    ⎩ 5)320
      ——————  s.  d.
   12)64(5 : 4 Facit.
```

And thus in like manner you may multiply by any vulgar fraction.

CASE III.

To reduce the different denominations of money weights, meaſures, &c. to their equivalent decimal values.

Rule.

To the number of parts of the leſſer denomination given, annex a competent number of cyphers,

phers, and divide by the number of ſuch parts that are contained in the greater denomination, to which the decimal is to be brought; and the quotient is the decimal required.

Examples.

What decimal of a pound, is 17 ſhillings?

2|0)17.0|0

Anſwer .85

Reduce 14s. 6d. to the decimal of a pound.

s. d.
20 14 : 6
12 12

24|0) 174.00|0(.725 Anſwer.
168

60
48

120
120

Let 3 $\frac{1}{4}$ inches be reduced to the decimal of a foot.

I.
12 3 $\frac{1}{4}$
4 4
48 (13.0000(.2708 Anſwer.
96
340
336
400
384
16

Reduce 2qr. 25lb. to the decimal of a hundred weight.

Qr. lb.
4 2 : 25
28 28
112) 81.00(.7232 Anſwer.
784
260
224
360
336
240
224
16

What decimal of a foot is 4 inches?
12)4.0
Anſwer .3

N. B. There is another rule which is ſomewhat ſhorter than the preceeding one; that is, Write the given denomination, or parts, orderly one under the other, the inferior or leaſt parts being uppermoſt; let theſe be dividends; againſt each part on the left-hand, write the number contained in one of its next ſuperior, and let theſe be diviſors: Then beginning with the upper one, write the quotient of each diviſion as decimal parts, on the right-hand of the dividend next below it; and let this mixt number be divided by its diviſor, &c. and the laſt quotient will be the decimal ſought.

EXAMPLES.

Reduce 8s. 4d. to its equivalent decimal of a pound ſterling.

12	4
20	$8.\dot{3}$

Thus $.41\dot{6}$ of a £. is equal to 8s. and 4d.

What decimal of a £. is equal to 15 s. 8d. $\frac{1}{4}$?

4	1
12	8.25
20	15.6875

Anſwer .784375£.

What decimal part of a foot is equal to 7 inches 5 parts?

12	5
12	$7.41\dot{6}$

Anſwer $.6180\dot{5}$ of a foot.

What's the decimal of 9 oz. 16 dwts. 10 gr. a lb. troy being the integer?

24 $\begin{cases}4\\6\end{cases}$	10 (2.5
20	$16.41\dot{6}$
12	$9.8208\dot{3}$

Anſwer $.818402\dot{7}$ lb. [troy.

It is of uſe in decimal computations, to have tables of the decimal values of the parts of coin, weights, meaſures, &c. therefore the following tables are calculated for that purpoſe.

And the manner of conſtructing theſe or any ſuch like tables, may be done either by the preceding rules, or by the following one: viz. The ſhillings 19, 18, 17, 16, &c. are ſeparately divided by 20, and the ſeveral quotients are the decimals of their reſpective ſhillings. Next the decimals of 11, 10, 9, &c. ſhillings, are divided by 12, and the quotients are the decimals of the pence 11, 10, 9, &c. Then the decimals of 3, 2, 1, pence, are divided by 4, and the quotients are the decimals of the farthings 3, 2, 1.

Alſo in the ſame manner are the other tables of weights, meaſures, &c. conſtructed, having a regard how many of a leſſer denomination are contained in a ſuperior one.

TABLE I.

COIN, *one pound the integer.*

Shillings.	Decimals.
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

Pence.	Decimals.
11	.04583
10	.0416
9	.0375
8	.03
7	.92916
6	.025
5	.02083
4	.016
3	.0125
2	.0083
1	.00416

Farthings.	Decimals.
3	.003125
2	.002083
1	.0010416

Note. The following table of ounces will ſerve for inches, and any thing where 12 is the integer.

TABLE II.

TROY WEIGHT, *one pound the integer.*

Ounces.	Decimals.	Dwts.	Decimals.
11	.916̇	10	.0416̇
10	.83̇	9	.0375
9	.75	8	.03̇
8	.6̇	7	.02916̇
7	.583̇	6	.025
6	.5	5	.02083̇
5	.416̇	4	.016̇
4	.3̇	3	.0125
3	.25	2	.0083̇
2	.16̇	1	.00416̇
1	.083̇		

Grains.	Decimals.
10	.001736
9	.001563
8	.001389
7	.001215
6	.001042
5	.000868
4	.000694
3	.000521
2	.000347
1	.000173

TABLE

TABLE III.

AVOIRDUPOIS WEIGHT.

1 *Cwt. the integer.*		1 *Pound the integer.*	
Pounds.	Decimals.	Ounces.	Decimals.
20	.178571	12	.75
10	.089285	10	.625
9	.080357	9	.5625
8	.071428	8	.5
7	.0625	7	.4375
6	.053571	6	.375
5	.044642	5	.3125
4	.035714	4	.25
3	.026785	3	.1875
2	.017857	2	.125
1	.008928	1	.0625
Ounces.	Decimals.	Drams.	Decimals.
10	.005580	10	.039062
9	.005022	9	.035156
8	.004464	8	.03125
7	.003906	7	.027344
6	.003348	6	.023437
5	.002709	5	.019531
4	.002232	4	.015625
3	.001674	3	.011718
2	.001116	2	.007812
1	.000558	1	.003906

Note. If you have occaſion for the decimals of a ton weight, the decimals of the ſhillings in the firſt table will ſerve for the hundred weights, ſuppoſing one ton the integer; and for quarters, .25, .5, and .75.

TABLE

TABLE IV.

LIQUID MEASURE.

1 *Hhd.* *the integer.*

Gall.	Decimals.
60	.952381
50	.793651
40	.634921
30	.47619
20	.31746
10	.15873
9	.142857
8	.126984
7	.1
6	.095238
5	.079365
4	.063492
3	.047619
2	.031746
1	.015873
Pints.	**Decimals**
7	.013889
6	.011905
5	.009921
4	.007937
3	.005952
2	.003968
1	.001984

TABLE V.

LONG MEASURE.

1 *Yard* *the integer.*

Feet.	Decimals.
1	.6
2	.3
Inch.	**Decim.**
11	.305
10	.27
9	.249
8	.2
7	.194
6	.16
5	.138
4	.1
3	.083
2	.05
1	.027
Qu. I.	**Decim.**
3	.02083
2	.0138
1	.00694

Note. When you want the decimal of a foot for any number of inches, you may make uſe of the decimal annexed to ſuch number of ounces in table II.

TABLE VI.

TIME.

An Year the integer.

Days.	Decimals.
300	.821918
200	.547945
100	.273973
90	.246575
80	.219178
70	.191781
60	.164383
50	.136986
40	.109589
30	.082192
20	.054794
10	.027397
9	.024657
8	.021918
7	.019178
6	.016438
5	.013698
4	.010959
3	.008219
2	.005479
1	.00274

N. B. In making uſe of the tables, if the number of parts wanted are not found therein, take it out at twice; thus, for 15 dwts. add 10 and 5 together; and the like for any other.

The

The uſe of the foregoing tables.

What decimal of a pound ſterling is 14s. and 10d. ½ ?

In TABLE I.

Againſt 14s. is .700000
Againſt 10d. is .041666
Againſt 0d. ½ is .002083

These added, their ſum is .74375, the decimal required.

What decimal part of a year is 200 days ?

Anſwer .547945

What decimal of a foot is 7 inches and 3 parts ?

12).25

.02083
.58333

Anſwer .60416

N.B. In making uſe of the ſecond table in this caſe, you muſt divide the decimal againſt 3 in the table by 12 for the parts, which added to the decimal of 7 for inches, gives the anſwer right.

What is the decimal of 1 foot 4 ½ inches, a yard being the integer ?

.3333
.1111
.0138

.4583 Anſwer.

A ſhort and eaſy method of reducing troy weight and avoirdupois weight into each other. Multiply the pounds (and decimal parts of a pound) avoirdupois by 1.21527, and it gives the like value in troy weight; and multiplying the pounds troy by .822857 $\frac{1}{7}$ gives its equivalent value in avoirdupois weight. Theſe factors are found by dividing the number of grains in a pound by each other.

EXAMPLES.

Reduce 202 $\frac{3}{4}$ lb. avoirdupois into troy.

```
          1.21527
           202.75
      -----------
           607638
          8506944
         24305555
       2430555555
      -----------    lb.  oz. dwts. gr.
Anſ.  246.3975694 or 246 : 4 : 15 : 10
```

Reduce 175 lb. troy into avoirdupois.

```
       .822857 1/7          7)175
               175          -----
       -----------             25
           4114310
          5759999
          822857
       -----------
Anſwer 144.000000
```

Note. The troy pound is to the avoirdupois pound in the ſame proportion as the wine gallon is to the ale gallon.

Or as there are 5760 grains contained in the pound troy, the avoirdupois pound conſiſts of 7000 grains, which two numbers are alſo proper factors to reduce theſe weights into each other; viz. if you would reduce troy pounds into avoirdupois pounds, multiply the numbers of pounds into the leſſer number and divide by the greater; obſerving the contrary for troy pounds. Otherwiſe 576 and 700; or 144 and 175 are factors which will anſwer the ſame purpoſe. But as the pound troy is leſs than the avoirdupois, yet the ounce is greater; therefore if you would reduce the ounces into each other, multiply the troy ounces by 56 and divide by 51 for avoirdupois ounces; and for troy ounces, multiply the avoirdupois ounces by 51 and divide by 56.

PROPORTION *or* The RULE OF THREE

IS when there are three numbers given to find out a fourth.

It is the rule of three Direct when the three given numbers (or terms) require a fourth number that ſhall bear the ſame proportion to the third as the ſecond does to the firſt; and here the ſecond and third terms are to be multiplied together, and their product divided by the firſt.

The proper method of ſtating queſtions in this rule, is thus,

As 4 : 12 : : 8 : 24.

viz. as 4 is to 12, ſo is 8 to 24.

It is called the rule of three Indirect when the 3 numbers given require a fourth that ſhall bear the ſame proportion to the ſecond as the third does to the firſt; and in this caſe the firſt and ſecond numbers are multiplied together, and their product divided by the third.

In ſtating all queſtions in proportion (whether they require a ſingle or a double ſtating) obſerve where the demand lies, and make that your third number, and ſee the firſt and third numbers are of one denomination, or ſo reduced; and your ſecond or middle number is always of the ſame name with the fourth number or anſwer required, and muſt be reduc'd into its loweſt denomination mentioned.

When a queſtion is ſtated, if the third number, being more than the firſt requires more, or, being leſs requires leſs, it is *direct*; but if the third number, being more, requires leſs, or being leſs, requires more, it is *indirect*.

N. B. The queſtions I have made choice of are promiſcuouſly inſerted, in order to ſtrengthen the judgment of the young practitioner; and which method of practice I apprehend to be the beſt.

EXAMPLES.

If I give 2 guineas for $3\frac{1}{2}$ yards, what quantity ſhall I have for 21l. 10s. at the ſame rate?

£. yds. £.
As 2.1 : 3.5 :: 21.5
3.5
———
1075
645
——— Yds.
2,1)75.25 (35.83 Anſwer
63
———
122
105
———
175
168
———
70
63
———
7

If my yearly income be 39 l. what is that per week?

w. £. w.
As 52 : 39 :: 1
20 s.
——— 15
52)780 Anſwer.
52
———
260
260
———

If a rule of 2 foot long give 3 ½ feet in ſhade, how high is that ſteeple which gives 43 yards in ſhade?

43
3
As 3.5 : 2 :: 129
2
F.
3.5)258.0(73.7 Anſwer.
245
130
105
250
245
5

If 42lb. coſt 17s. 6d. what is that per lb?

As 42 : 17.5 :: 1
42)17.5(.416
168 12
70 5.000
42
Anſwer 5d.
280
252
28

How many yards of matting of ½ yard wide, will be ſufficient to cover a floor that is 16 feet wide, and 28 feet long?

```
As 16  :  28  : :  1.5
          16
         ----
         168
         28
      ---------    3)
 1.5)448.0(     298.6
       30        -----
      ----       99.5 yds.
      148         Anſwer.
      135       ----------
      ----
       130
       120
      -----
        100
         90
        ----
         10
```

What is the intereſt of 320 l. at 4 ½ per cent. per annum?

```
                    As 100 : 4.5 : : 320
                                     4.5
                                  -------
                                   1600
                                  1280
                                  -------
                              1|00)14|40.0
                                      20
                                  -------
Anſwer 14l. 8s.                    8.00
```

If 112 lb. weight at London make 99 lb. at Lisbon, how many lb. at London are equal to 1049 lb. at Lisbon?

As 99 : 112 :: 1049
112
2098
1049
1049
99)117488(1186.74 lb. Anſwer.
99
184
99
858
792
668
594
740
693
470
396
74

If 75l. is put to intereſt, and at the end of 12 months it amounts to 81l. at what rate per cent. was the intereſt allow'd?

81
75
6

As 75 : 6 :: 100
6
75)600(8l. Anſwer.
600

An acre of land contains 40 perches in length, and 4 in breadth; what muſt be the length to make an acre when the breadth is but 15 yards?

5.5
4
22.0

As 22 : 220 :: 15
22
440
440

5.5
40
220.0

15)4840(Yards 322.6 or yds. 322 f. 2 Anſwer.
45
34
30
40
30
100
90
10

If a man can do a piece of work in 6 days when the day is 8 hours long; in what time will the ſame be done when the day is 10 hours long?

As 8 : 6 :: 10
6
10)48
d. 4.8
24
h. 19.2
60
m. 12.0 Anſwer D. 4 H. 19 M. 12

If when the buſhel of wheat coſts 3s. 4d. the penny loaf weighs 12 ounces; what will be the weight of a 9 penny loaf (at the ſame rate) when the buſhel coſts 4s. 4d?

```
   s. d.   oz.    s. d.
As 3  4 : 12 :: 4  4
   12           12
   ----         ----   d.    oz.      d.
   40           52   As 1 : 9.23 :: 9
   12                              9
   ----                     --------
52)480(9.23                 16)83.07(5.19
   468                         80     16
   ---                         --     ---
    120   lb. oz.               30    114
    104    5  3                 16    19
    ---    Anſwer.              ---   ---
     160                        147   3.04
     156                        144
     ---                        ---
       4                          3
```

In 145l. 12s. ſterling, how much flemiſh money at the rate of 1l. 13s. 4d. to the pound ſterling?

```
          £.  s. d.    £.
      As  1 : 33  4 :: 145.6
              12        400
             ----   -----------
             400   12)58240.0
                   -----------
                   2|0)485|3   4
                   -------------
           Anſwer £. 242 13 4
```

Bought a quantity of braſs at 4l. 6s. per C. how muſt I ſell it to gain 5 per cent.?

As 100 : 4.3 :: 105
4.3

315
420

1|00)4|51.5
20

10.300
12

3.6
4

2.4

£. s. d.
4 10 3 ½ per C.
Anſwer.

Suppoſe 75 gallons of water do in one hour fall into a ciſtern containing 250 gallons, and by a pipe in the ſame ciſtern there runs out at the ſame time 51 gallons in the hour; in how many hours will it be filled?

75
51

24

As 24 : 1 :: 250
24) 250 (10.416
24 60

100 25.000
96

40
24

160
144

16

H. M.
Anſwer 10 25

Suppoſe I deliver to a refiner a quantity of waſte metal, averdupois weight, in order to have it refined, and he agrees to return 95lb. of 100 troy weight; what factor ſhall be found to multiply the given weight averdupois into the return'd pounds troy, at the rate of allowance for waſte, &c. of 5 lb. in the 100 troy?

N. B. I have inſerted in page (35) the proper factors for reducing avoirdupois weight and troy weight into each other. Now as 1.21527 is the factor for reducing avoirdupois pounds into troy, the number required is found by this proportion; As 1 integer is to 1.21527, ſo is the .95 decimal of an integer to the factor ſought for: See the work.

```
As 1 : 1.21527 : : .95
                    .95
              ---------
                 607638
               1093750
              ---------
      Anſwer 1.1545138
```

A borrow'd of B 40 l. for 6 months; afterwards B borrow'd of A 60 l. how long ought he to keep it?

```
        As 40 : 6 : : 60
            6
         ------
   6|0)24|0
       ------
   Anſwer 4 months.
```

If by laying out a ſum of money 1s. brings 14d ¾, what is gained per cent.?

s. d. £.
As 1 : 2¾ :: 100
20

2000
2.75

12)5500.00

2|0)45|8.(4

£. s. d.
Anſwer 22 18 4

Bought a quantity of old ſilver for 8l. 1s. which weighed 3 ½ lb. troy; what did the ſame lie me in per ounce?

lb.oz. £. s. oz.
As 3 6 : 8 1 :: 1
12 20

42) 161 (3.83
126 12

350 10.00
336

140
126

14

s. d.
Anſwer 3 10

If 100 l. principal gain 5 l. intereſt in 12 months, what principal will gain as much in 5 months?

M. £. M.
As 12 : 100 :: 5
12

5)1200

Anſwer 240 £.

Suppoſe I ſee a flaſh of lightning *, and count 6 ſeconds before I hear the thunder, how far is the thunder cloud from me ?

N. B. It is well known both by theory and experiments, that ſound flies at the rate of 1142 feet per ſecond.

S. F. S.
As 1 : 1142 :: 6
6
Mile yds.
1760 yds. in a mile. 6852 feet or 1 and 524
3 5280 Anſwer.
5280 3)1572
524

---

* From Mr. Franklin's experiments, it appears, that Lightning is only electrical fire drawn off from the clouds; and this electrical fire has in reality been collected from the clouds during thunder, by means of iron bars or tin tubes, in many parts of Europe, but more particularly in America where Mr. Franklin made his experiments: Therefore it were much to be wiſhed that Mr. Franklin's bars were more generally uſed than they are, for the preſervation not only of public and private buildings, but alſo for the preſervation of the lives of men and beaſts; as Mr. Franklin has made appear in his experiments; to which work I refer the reader, for a more particular account.

PRACTICE,

IN general, is a ſhort way of finding the value of any quantity by the given price of one integer; but to give all the diverſity of methods that are applicable to the rules of practice, wou'd be an endleſs task; therefore it may ſuffice in this place to take notice of the moſt general of thoſe rules; which is, to divide the quantity by ſuch aliquot part or parts as is the given price, &c. Alſo the ſame methods muſt be taken in working of decimal parts as in whole numbers.

A TABLE OF ALIQUOT PARTS.

Parts of a Pound. s. d.			Parts of a Shilling.			Parts of C. wt.	
10 0	is	$\frac{1}{2}$	6d.	is	$\frac{1}{2}$		
6 8	is	$\frac{1}{3}$	4d.	is	$\frac{1}{3}$	56lb.	$\frac{1}{2}$
5 0	is	$\frac{1}{4}$	3d.	is	$\frac{1}{4}$	28	$\frac{1}{4}$
4 0	is	$\frac{1}{5}$	2d.	is	$\frac{1}{6}$	16	$\frac{1}{7}$
3 4	is	$\frac{1}{6}$	$1\frac{1}{2}$d.	is	$\frac{1}{8}$	14	$\frac{1}{8}$
2 6	is	$\frac{1}{8}$	1d.	is	$\frac{1}{12}$	8	$\frac{1}{14}$
2 0	is	$\frac{1}{10}$	$\frac{3}{4}$d.	is	$\frac{1}{16}$	7	$\frac{1}{16}$

EXAMPLES.

At 3s. 3d. per lb. what will 58lb. come to?

2s.	$\frac{1}{10}$	58		2s.	$\frac{1}{10}$	58
1s.	$\frac{1}{2}$	5 16		1s.	$\frac{1}{2}$	5.8
3d.	$\frac{1}{4}$	2 18		3d.	$\frac{1}{4}$	2.9
		0 14 6				0.725

Anſwer £ 9 8 6 or thus £ 9.425 decimally.

At 5 s. 4 d. ½ per yard, what will 222 yards come to?

4s.	⅕	222
1s.	¼	44.4
4d.	⅓	11.1
½d.	⅛	3.7
		.4625

£. s. d.
Anſwer 59.6625 or 59 13 3

At 5 d. ½ per oz. what will 360 ¾ ounces come to?

4d.	⅓	360.75
1½d.	⅛	120.25
		45.09375

20)165.34375

£. s. d.
Anſwer 8.267, &c. or 8 5 4

At 1s. 9d. per ell, what will 387 yards come to at that rate?

d.		
6	½	309.6
3	½	154.8
		77.4

2|0)54|1.8

£. 27 1 9½.4
Anſwer.

4
5)1548
309⅗

5)3.0
.6

.8
12
d. 9.6
4
f. 2.4

At 4l. 13s. 6d. per C. wt. what will 425 C. 3 Q. 9 lb. come to?

		£.	s.	d.				
		0	10	0	½	425		
						4		
		4	0	0		1700		
		0	2	0	1/5	212	10	
		0	1	0	½	42	10	
		0	0	6	½	21	5	
						10	12	6
2qr.	½	4	13	6				
1qr.	½	-	-	-	- - -	2	6	9
7lb.	¼	-	-	-	- - -	1	3	4 ½
1lb.	1/7	-	-	-	- - -	0	5	10+.5
1lb.		-	-	-	- -		0	10—.07, &c.
								10—.07, &c.
						£.	s.	d.
		Answer				1990	15	1 ½.64, &c.

FELLOWSHIP or COMPANY.

Without taking any notice of the common way of working fellowſhip, obſerve this more compendious rule: Divide the whole gain or loſs, by the whole ſtock, and multiply the quotient by each perſon's particular ſtock; the ſeveral products are the reſpective gains of each.

By this rule, bankrupts eſtates, legacies, &c. are adjuſted, in caſe of a deficiency in effects, &c.

Note. If there be any ſhillings, &c. in any one's ſtock, you muſt firſt reduce it to the decimal of a pound, and proceed on as aforeſaid. And in queſtions with time, you muſt multiply each particular ſtock by its reſpective time, and add all the products together; and in the rule, call it your whole ſtock, &c.

EXAMPLES.

A, B, and C, trading together; A, puts into ſtock 500l. B, puts 700l. and C, puts in 1200l. and the whole gain is 836l; what ſhare of it belongs to each?

```
                              £.
                             500
                             700
                            1200
                            ----
                            2400
24|00)836|.00  (.3483                .3483
      72              500             1200
      --        ----------         ----------
      116       174.16 ..           6966 ..
       96       A's ſhare.         34833
      --        ----------         ----------
      200                           418.00 ..
      192                          C's ſhare.
      --          .3483            ----------
       80            700
       72       ----------                £.
       --       243.83 ..          A.  174 16
        8       B's ſhare.         B.  243.83
                ----------         C.  418.00
                                   ----------
                               Proof   836.00
                               --------------
```

A legacy being left as follows, *viz.* to A. the value of 500l. between B and C 350l, and to D 100l.; but by loſſes before the dividends could be paid off, the legacy ſuffer'd 250l. what does each dividend come to bearing a part proportionable in the loſs?

```
500
350     950)700.000 ( .73684          .73684
100         6650              500          350
----       --------   -----------     ---------
950          3500     368.42000        3684200
250          2850     A.       20      221052
----        ------          ------     ---------
700          6500            8.4       257.89400
             5700            12      B.&C.   20
            ------          ------       -------
              8000           4.8          17.88
              7600           4              12
             ------         ------       -------
               4000          3.2          10.56
               3800                          4
              ------                     -------
                200                        2.24
```

```
        Anſwer.                             .73684
                   £.   s.  d.                 100
A's ſhare         368   8   4¾             ---------
B & C's ſhare     257  17   10½            73.68400
D's ſhare          73  13   8              D.     20
                            ¾ rem.         ---------
                 --------------              13.68
     Proof        700   0   0                   12
                 --------------              -------
                                              8.16
```

A put into company 525l. 10s. for 6 months, B, 382l. 15s. for 8 months, and C, 1000l. for 4 months; they gain in all 1286l. 12s.; how much is that for each?

525.5	382.75	1000
6	8	4
3153.0	3062.00	4000

3153.0
3062
4000

10215)1286.600000(.125952
10215
26510
20430
60800
51075
97250
91935
53150
51075
20750
20430
320

.125952
3153
377856
629760
125952
377856
A. 397.126656

.125952
3062
251904
755712
3778560
B. 385.665024

.125952
.4000
C. 503.808000

A's ſhare	397.126656
B's ſhare	385.665024
C's ſhare	503.808
Total	1286.599680
Or	1286l. 12s. very near.

Suppoſe a bankrupt is indebted 2050l. 10s. and his effects amounts but to 715l. 5s. what is that in the pound?

2050.5)715.25(.3488
61515

100100
82020

180800
164040

167600
164040

3560

.3488
20

6.9760
12

11.7120
4

2.8480

Anſwer 6s. 11d. ½

Three merchants A B and C freight a ſhip with 96 ton of wine: Thus A loaded 24 ton, B 32, and C 40 ton; but by extremity of weather they caſt 12 ton overboard; how much muſt each merchant bear of this loſs?

24
32
40

96

96)12.000(.125
96

240
192

480
480

.125
24

500
250

Ton 3.000
A's loſs.

.125
32

250
375

B's loſs 4.000

.125
40

5.000
C's loſs.

Suppoſe 20 ſhillings to be ſhared among 3 perſons; A, to have ½; B, ⅓; and C, ¼: what is the ſhare of each in proportion one with another?

```
                           s.       s.          s.
 .5                      ½)20     ⅓)20        ¼)20
 .3                        10      6.6         5.0
 .25             s.                 6           5
 1.083           10                6.0         4.5
  108            10
                       s.
 .975)          9.000 (.230769
                8775   A's ſhare.
                                       s.     s.
                2250          .975)6.000 ( 6.153846
                1950                5850     B's ſhare.
                3000                1500
                2925                 975
                 7500                5250
                 6825                4875
                  6750                3750
                  5850                2925
                   9000                8250
                   8775                7800
                    225                 4500
                                        3900
                                         6000
                                         5850
                                          150
```

See the remainder of the work in the next page.

```
.975)4.500(4.615384                      225 }
     3900  (C's ſhare.                   150 } remainders.
     ----                                600 }
     6000                                ---
     5850                                975
     ----                                ---
      1500
       975
      ----                        s.
      5250                        9.230769
      4875                        6.153846
      ----                        4.615384
       3750                        .000001 for the re-
       2925                       ---------        mainder.
       ----             Proof    20.000000
        8250                     ---------
        7800
        ----
         4500
         3900
         ----
          600
```

Three merchants being in partnerſhip 16 months: A put into common ſtock 100l. and at 8 months end took out 40l. and at 4 months after put in 140l. B put in at firſt 200l. and at 6 months end put in 50l. more, and at 4 months after took out 150l. C put in at firſt 150l. and at 4 months end took out 50l. and at 8 months after put in 100l. and they gained 357l. What muſt each perſon have proportionable to his time and ſtock?

See the work in the next page.

100 200 150
8 6 4

800 1200 600

60 250 100
4 4 8

240 1000 800

200 100 200
4 6 4

800 600 800
240 1000 800
800 1200 600

1840 2800 2200
2800
2200

6840 total ſum of the products.

684|0)357.00000|0(.052192
3420

1500
1368

1320
684

6360
6156

2040
1368

672

.052192
1840

2087680
417536
52192

A. 96.033280

.052192
2800

41753600
104384

B. 146.137600

.052192
2200

10438400
104384

C. 114.822400

£.
96.03328 A's ſhare.
146.1376 B's ſhare.
114.8224 C's ſhare.

356.99328 total or very near.

SIMPLE INTEREST.

Intereſt in general may be work'd by the rule of three; but there are ſpeedier ways to perform the ſame by proper factors, and other cuſtomary methods.

The annual intereſt of any ſum of money is found by only multiplying the given principal by the intereſt of one pound for a year, which at 4 per cent. the ratio is .04; at 4 ½ per cent. the rate is .045; at 5 per cent. the ratio or intereſt is .05, &c. found by dividing the rate of intereſt by 100. Thus you may find ratios at what rate per cent. you pleaſe.

Examples.

What's a year's intereſt of 536 l. at 5 per cent.?

.05

£. s.

Anſwer 26.80 or 26 16

Thus a year's intereſt of any ſum being found; 'tis only multiplying that ſum by 2, 3, 4, &c. and the product is the intereſt of the ſaid ſum for 2, 3, 4, &c. years.

What's the intereſt of 623 l. for 3 years, at 4 per cent. per annum?

623

.04

24.92

3

£. s. d.

Anſwer 74.76 or 74 15 2 ¼

Again, if the time required be months, 'tis only dividing the year's amount by ſuch aliquot parts according to the rules of practice as the given months are of an year.

What's the intereſt of 539 l. for 4 months, at 4 ½ per cent. per annum?

539
.045

2695
2156

M.
4 ⅓ 24.255

8.085 Anſwer £. 8 s. 1 d. 8 ¼

What will the intereſt of 729l. 10s. 6d. come to in a year and 5 months, at 5 per cent. per annum?

12 6
20 10.5

.525

729.525
.05

M.
3 ¼ 36.47625
2 ⅔ 9.11906
6.07937

Anſwer 51.67468 or £. 51 s. 13 d. 5 ¾

But if it be required to find the intereſt of a ſum of money for any number of days, you may either multiply the year's amount by the number of days propoſed, and divide the product by 365, &c. or (which will be ſometimes ſhorter) divide the year's amount by 365, and multiply the quotient by the number of days propoſed.

What intereſt will 563 l. amount to in 126 days, at 5 per cent. per annum?

```
   £.                     126
   563                    .077
   .05                   -----
 -------                  882
365)28.15(.077            882
    2555                 -----
   ------                 9.702
    2600                 -----      £. s. d.
    2555                 Anſwer     9 14 0 ½
   ------                -------------------
     45
```

N. B. There is another ſhort and cuſtomary method of working ſimple intereſt: 'Tis done by multiplying the principal or given ſum by 2, 3, 4, or 5, &c. according to the rate per cent. and cut off two figures towards the right hand, which is the ſame as multiplying by the other ratios, (or dividing by 100;) then multiply the figures cut off to the right hand by 20, and take in the odd ſhillings, and cut off two figures as before; the like muſt be done by multiplying the remainder by 12 for pence, taking in the odd pence, and cutting off as before, &c.

EXAMPLES.

What is the intereſt of 275l. 10s. for a year, at 5 per cent.?

```
                  5
     -----------------
£.     13|77 10
          20
     -----------------
s.     15|50
          12
     -----------------      £.  s. d.
d.      6|00    Anſ.  13  15  6
```

Note. By this method, merchants and traders eſtimate their allowances upon ſome ſorts of goods, which is called the diſcount on ſuch goods. For example, ſuppoſe I buy 20 dozen of buckles at 5s. per doz. with an allowance of 10 per cent. diſcount from the maker; and I buy 20 doz. of the ſame of another maker at 5s. and 6d. per doz. with the diſcount of 20 per cent. which do I buy cheapeſt? The anſwer produces 2s. gained in the 20 dozen at 5s. and 6d.

When the given rate per cent. is not even pounds, but is 3½, 4¾, or the like, then multiply the principal or given ſum by the even pounds, and take the ½, or ¼, or ¾ of the ſaid principal or given ſum and add to that product; then cut off two figures, and proceed as before.

What comes the intereſt of 585l. 2s. 6d. to, at 4½ per cent.?

```
            585   2   6
                      4
 s.  |   | ------------
 10  | ½ | 2340  10   0
            292  11   3
           ------------
 £.      26|33    1   3
             20
          -----
 s.       6|61
             12
          -----                £.  s. d.
 d.       7|35           Anſ.  26  6  7¼
              4
          -----
 q.       1|40
          -----
```

Lent 410l. upon a mortgage, to receive intereſt at $4\frac{3}{4}$ per cent. till it was paid off; and the mortgage being paid off 3 years 8 months after; what was the intereſt upon the ſame?

```
            £.                          £.   s.  d.
           410                          19   9   6
             4                                   3
  s.      ————           M.|   |      ————————————
 10|½|   1640             6| ½ |      58    8   6
  5|½|    205   s.        2| ⅓ |       9   14   9
     |    102   10             |       3    4  11
        ————                          ————————————
 £.    19|47                   £.     71    8   2
          20                          Anſwer.
        ————
        9|50
          12
        ————
        6|00
```

Here you alſo ſee the manner of taking in the even parts of a year; but if the intereſt required is for ſuch a number of days as cannot well be taken in their even parts, firſt find the intereſt of one year as before; and for the days, you may work them by proportion; *viz.* As 365 days is to the year's amount, ſo is ſo many days to the anſwer requir'd.

Suppoſe the intereſt of one year is 12l. 5s. 6d. what will be the intereſt of 69 days at that rate?

```
    D.   £. s. d.    D.
As 365 : 12  5  6 :: 69               12 | 6
         20                           20 | 5.5
        ----                               -----
        245                                .275
         12
       -----
       2946
         69               Or thus decimally.
      ------              As 365 : 12.275 :: 69
      26514                            69
     17676          12)              -------
    --------       (556   ¾           110475
365)203274        ----------          73650       £.
    1825          2|0)4|6  4       365)846.975(2.320
    -----                              730         Anſ.
    2077          ------------        ------
    1825          £ 2  6  4¾           1169
    -----         Anſwer.              1095
    2524                               -----
    2190                                747
    -----                               730
     334                                ----
       4                                175
    -----
365)1336 (3
    1095
    -----
     241
```

But if we ſuppoſe the principal to be 1, the rate 1, and the time 1, we can eaſily conſtruct a ſmall yet univerſal table for any principal, any number of days, or for any rate of intereſt: Thus if the principal, time and rate are 1 each, theſe multiplied together produce 1; which divided by 365 gives .002739726, which gives the firſt number in the following table; and if it is multiplied by 2, 3, 4, 5, &c. it will give the other numbers. Hence it is plain from the conſtruction of the table, that if any principal, time and rate is multiplied together, and the product taken out of the table, that will be the intereſt required for the given ſum, time, and rate of intereſt.

TABLE.

1	2739726
2	5479452
3	8219178
4	10958904
5	13698630
6	16438356
7	19178082
8	21917808
9	24657534
10	27397260

Note. If the product hath

figures,	point off decimals
7	5
6	6
5	7
4	8
3	9
2	10
1	11

EXAMPLES.

What is the intereſt of 110l. for 8 days at 5 per cent.?

110	4000 = .10958904
40 *	400 = .010958904
4400	Anſwer .120547944 or 2s. 4d ¾

* Here I multiply by 40, which is the ſame thing as if I multiplied by 8 and by 5.

What is the intereſt of 160l. for 144 days at 5 per cent.?

144
160
8640
144
23040
5
115200

100000 = 2.739726
10000 = .2739726
5000 = .13698630
200 = .005479452
115200 = 3.156164352 Anſ.

But notwithſtanding what has been ſaid on ſimple intereſt for days or any part leſs than an year, the true intereſt ſhould be reckoned according to ſuch progreſſive order as compound intereſt for ſuch time; the true time of ſimple intereſt being limited to one year only. Therefore if a man pays intereſt for 100l. half yearly at 5 per cent. he ſhould not pay 2l. 10s. as is generally ſuppoſed, but £. 2.4695, that is 2l. 9s. 4d. $\frac{1}{2}$ .72. And if the intereſt be paid in leſs than half an year (as aforeſaid) the error will be greater; to demonſtrate which, obſerve what follows.

It is well known that the intereſt or amount of one pound for any time greater than an year is found by raiſing the amount of one pound to the power of the time; ſo for any part of an year it is found by extracting the root of the power of the time.

A Table ſhewing the amount of 1l. from 1 month to a year.

Months	£.
1	1.004074
2	1.008169
3	1.012272
4	1.016390
5	1.020537
6	1.024695
7	1.028870
8	1.033061
9	1.037270
10	1.041496
11	1.045740
12	1.050000

For example, if you wou'd know how much a perſon ſhould pay quarterly for intereſt of 100 l. at 5 per cent. look in the table for 3 months, and againſt it you will find 1.012272, which multiplied by 100 becomes 101.2272 for the amount of 100l. in ¼ of an year, and the intereſt is 1.2272 or 1l. 4s. 6d. ½ inſtead of £. 1.25 or 1l. 5s.; the error in this caſe being 5d. ½ for ¼ of an year. Therefore if a man pays intereſt for money quarterly he ought to have 22d. per cent. per annum abated him; and if he pays it half yearly he ought to have 14d. ¾ or 15d. per cent. per annum abated him. Nevertheleſs an error in this caſe (its boundaries being limited within the year) is very inconſiderable to that common error of reckoning diſcount like intereſt.

REBATE OR DISCOUNT,

ACcording to ſimple intereſt, is to abate in receiving money, as much as the intereſt comes to, for the time 'tis paid before it becomes due. The reaſon of this is evident from the following propoſition.

Suppoſe 100l. preſent money would diſcharge a debt of 105l due a year to come, rebate being made at 5 per cent. becauſe 100l. being put to intereſt for a year at the ſaid rate wou'd regain the 5l.

To work queſtions in this rule, obſerve this proportion; As 100l. with the intereſt for the time, is to the intereſt alone, if you wou'd find the rebate, (or to 100l. if you would find the preſent money;) ſo is the debt or ſum propounded, to the anſwer required.

Example. I have a bill of 21 l. payable an year hence; what is the worth of it in preſent money if rebate is allowed at 5 per cent.?

As 105 : 100 : : 21
21

105)2100(20 £.
210 Anſwer.

Example. Sold goods to the value of 73l. 5s, to be paid in a year's time; what muſt be diſcounted for the preſent payment, if rebate be allowed at 4 ½ per cent?

```
     £.      £.      £.
As 104.5 : 4.5 :: 73.25
                    4.5
                 ------
                  36625
                 29300
                 ------
     104.5)329.625 (3.1543
           3135         20
          -----   --------
            1612    3.0860
            1045        12
          ------  --------
             5675   1.0320
             5225
            -----
              4500
              4180
             -----
              3200              £.  s.  d.
              3135     Anſwer   3   3   1
             -----
                65
```

Now the preſent practice of diſcounting bills, &c. is done by the intereſt of the whole ſum for ſo long a time; but ſuch ſum cannot be eſteem'd a principal, nor in full value, till the time of payment is expir'd; therefore leſs intereſt muſt be requir'd according to the juſt rule of diſcount or rebate on ſuch payment, which will be the true intereſt, and anſwers to the propoſition.

Example. What preſent money will diſcharge a debt of 100 l. payable at the end of 12 months, rebate being made at 5 per cent.?

```
As 105 : 100 :: 100
                100
   ------------
   105)10000 ( 95.238
        945         20
        ----    ------
        550     4.760
        525        12
        ----    -----
         250    9.120
         210
         ----
          400
          315             £.  s.  d.
          ----     Anſwer 95  4   9
           850
           840
          ----
            10
```

And here in this laſt example it will be obvious to any one's underſtanding, that to reckon diſcount like intereſt, is falſe and erroneous; for by the preſent practical method there would have been 5l. deducted from the 100 l. inſtead of 4l. 15s. 3d, which is 4s. 9d. too much in the 100l.

Note. Tho' it is with intereſt, that after having found the intereſt for 12 months, the half of that ſum may do for 6 months, and the 4th for 3 months, (the error in that caſe being but ſmall), but in rebate it is quite otherwiſe.

Again, for inſtance, the rebate of 500l. for a year, preſent payment, at 5 per cent. per annum, comes to no more than 23l 16s. 2d. ½, whereas in common practice, 25l. would be taken for the preſent payment of that ſum due a year hence; which fallacy is very evident, for the 25l. is the intereſt of 500l. (preſent worth or principal) for a year, and is a different thing from diſcount. In ſhort, by the preſent cuſtom of diſcounting ſums of money, &c. he that advances the money gets in effect the intereſt of what the diſcount falls ſhort of intereſt more than the rebate comes to. Thus upon tryal you'll find it very apparent, that the diſcount cannot be 25l. for then the preſent money would be but 475l. whereas it is 476l. 3s. 9d. ½, the intereſt of which for 12 months at 5 per cent. is 23l. 16s. 2d. ½ (very near,) both added together makes 500l. Whence it is evident, he who allows intereſt for diſcount wrongs himſelf conſiderably, which yet is very common among traders; for ſo much money ought to be paid, as at intereſt, would amount to the ſum due in the time propoſed.

Example. What preſent money is equal to 95l. due 9 months hence, at the rebate of 5 per cent.?

See the work in the next page.

£. £. £.
As 103.75 : 100 :: 95
100

M. | | £.
6 | ½ | 5
3 | ½ | 2.5
1.25

3.75
100

103.75

103.75)9500.00 (91.566
93375

16250
10375

58750
51875

68750
62250

65000
62250

2750

20

11.320
12

3.840
4

3.360

£. s. d.
Answer 91 11 3 ¾

EXAMPLE. What is the preſent value of a 10l. bill due 4 months hence, diſcounted at 4 per cent?

M. | | £.
4 | ⅓ | 4

1.3
100

101.3

As 101.3 : 100 :: 10
101 10

91.2 1000
1000

900.0

Thus the terms are prepared for the dividing part, there being a repetend in the diviſor; and the anſwer will produce 9l. 17s. 4d. ½ (very near) for the preſent payment of 10l. due 4 months to come, diſcounted at 4 per cent.

Example. What diſcount muſt be allow'd on a bill of 94l. 10s. payable in 98 days; rebate being made at 4 ½ per cent.?

D. £. D.
As 365 : 4.5 :: 98
4.5
490
392
365)441.0(1.2082
365
760
730
3000
2920
800
730
70

£. £. £.
As 101.2082 : 1.2082 :: 94.5
94.5
60410
48328
108738
101.2082)114.17490(1.128
1012082
1296670
1012082
2845880
2024164
8217160
8096656
120504

1.128
20
2.560
12
6.720
4
2.880

Anſwer £. 1 s. 2 d. 6 ½

If

Example. If 850l. is due at 1 year and 4 months to come, what muſt be diſcounted for the preſent payment, if rebate be allowed at 5 per cent.?

```
M. |   | £.        As 106.66 : 6.66 : : 850
 4 | ⅓ | 5                        850
       | 1.66                   -------
       -----                     33300
        6.66                    5328
        100                     -------
       ------         106.66)5661.00 ( 53.075
       106.66                5333O          20
                             ------      -------
                             32800        1.500
                             31998           12
                             ------      -------
                              80200       6.000
                              74662
                              ------
                               55380
Anſwer                         53330
£.  s.  d.                     ------
53  1   6                      2050
```

N. B. In this laſt example I have taken in but two of the circulating decimals, and ſo work'd as in common, in order to compare it to the ſame done by the rules of repetends which brings it to the exact truth; but if the repeating figures were carried to 6 or 7 places of decimals (tho' more tedious) it would bring it to very near the ſame as by obſerving the rules of repetends.

The laſt example work'd by repetends.

```
As 106 : 6 :: 850
     10       6
    ----    ----
     96   9)5100
          -----
           5666
            5666
          ------
      96)5100.0( 53.125
          480        20
         -----     -------
          300       2,500
          288          12
         -----     -------
           120      6.000
            96     -------
         -------    £.  s.  d.
            240     53  2   6
            192    True anſwer.
          -------
             480
             480
            -----
```

It is evident by a preceding example, that when diſcount is allowed for days the operation is long and tedious, and perhaps may be one reaſon why tradeſmen uſe intereſt for diſcount; but to remedy this inconveniency we have inſerted the following table, which ſhews the diſcount of any ſum for any number of days, at 3, 4, 5, 6, &c. per cent. intereſt.

	3	4	5	6	7	8	9	10
1	797978	1053741	1304631	1550788	1792344	2029427	2262159	2490660
2	1595957	2107482	2609263	3101577	2584688	4058853	4524318	4981320
3	2393935	3161222	3913904	4652365	5377032	6088280	6786477	7471980
4	3191914	4214963	5218526	6203153	7169376	8117707	9048636	9962640
5	3989892	5268704	6523157	7753942	8961721	10147133	11310795	12453300
6	4787871	6322445	7827789	9304730	10754065	12176560	13572955	14943960
7	5585849	7376185	9132420	10855518	12546409	14205987	15835114	17434620
8	6383828	8429926	10437052	12406307	14338753	16235414	18097273	19925280
9	7181806	9483667	11741683	13957095	16131097	18264840	20359432	22415940
10	7979785	10537408	13046314	15507883	17923441	20294267	22621591	24906600

Rule. Multiply the principal ſum by the number of days, ſeek for the product in the table and that will be the diſcount required.

Note. If the product of the days and principal hath

7 figures, point off 4 decimals.		
6 figures, point off 5 decimals.		
5 figures, point off 6 decimals.		
4 figures, point off 7 decimals.		
3 figures, point off 8 decimals.		
2 figures, point off 9 decimals.		
1 figures, point off 10 decimals.		

EXAMPLE.

What is the diſcount of 160l. for 144 days at 5 per cent.?

144
160

8640
144

23040

20000 = 2.609263
3000 = .3913904
40 = .005218526

3.005871926

£. s. d.
Diſcount required 3 0 1¼.637

Note. By the way, the aforementioned remarks on diſcount, naturally leads me to make ſome brief conſtructions upon another erroneous method much after the ſame manner, tho' not ſo much in uſe, which is in equation of payments. The deſign of this rule is, when ſeveral ſums are due at ſeveral times, to find a mean or equated time for paying the whole debt, ſo that neither debtor or creditor may ſuſtain loſs. Now the cuſtomary rule is, to multiply each ſum by its reſpective time, and add all the products together; then divide the total by the whole debt, and the quotient is called the mean time of paying the whole debt.—But to find the juſt mean or equated time of payment, you muſt firſt find out the preſent payment of every particular ſum in the queſtion payable at a time to come, by rebating at the rate of intereſt agreed on; then find in what time the ſum of thoſe preſent worths will be augmented to the total of all the particular ſums payable at times to come according to the firſt agreement; ſo ſhall the time found out, be the mean time for paying of the whole debt.—As equation of payments is very rarely made uſe of, I will leave the rule with this remark, ſo that you may prove the truth of it at your leaſure.

COMPOUND INTEREST,

OR

INTEREST UPON INTEREST,

IS that which ariſes from the principal and intereſt; that is, when intereſt on money becoming due and not paid, the ſame rate is allow'd on the unpaid intereſt as was before on the principal; ſo that the intereſt of the principal, and the intereſt of the intereſt added together, is the compound intereſt.

Rule. Multiply the principal by the rate per cent. which gives the intereſt for one year, this add to the principal, the ſum is the amount of the firſt year: Then this multiplied by the rate gives the intereſt for the ſecond year, which added to the firſt year's amount, gives the amount for the ſecond year, &c. &c.

Example. What will 251l. amount to, if it be forborn 3 years, at the rate of 4 ½ per cent. per annum, compound intereſt.

See the work in the next page.

251
.045
1255
1004
11.295
251
1ſt year's amount 262.295
.045
1311475
1049180
11.803275
262.295
2d amount 274.098275
.045
1370491375
1096393100
12.334422375
274.098275
£. s. d.
Anſwer 286.432697375 or 286 8 7¾

But the ſpeedieſt method to work queſtions in compound intereſt is by tables; therefore the following may ſerve as a ſpecimen.

A TABLE ſhewing the amount of one pound for years, at 5 per cent. per annum compound intereſt.

Years.	£.
1	1.050000
2	1.102500
3	1.157625
4	1.215506
5	1.276282
6	1.340096
7	1.407100
8	1.477455
9	1.551328
10	1.628895
11	1.710339
12	1.795856
13	1.885649
14	1.979932
15	2.078928
16	2.182875
17	2.292018
18	2.406619
19	2.526950
20	2.653298

&c.

The method to calculate tables of compound intereſt is to raiſe the amount of 1l. to the power of the time. Thus, at 5 per cent. the amount of one pound for one year (*viz.* 1.05) being multiply'd into itſelf or raiſ'd to the 2d power, gives the amount of 1l. for 2 years; and that amount being multiplied into the 1ſt year's amount gives the amount for the next year; and ſo on with any other rate of compound intereſt you begin your table with. THE

THE USE OF THE TABLE.

CASE I. To find the amount of any principal or ſum, according to the rate given at compound intereſt.

RULE. Multiply the ſum againſt the number of years propos'd by the principal, and the product will be the amount.

EXAMPLE. What will 225l. amount to in 3 years, at 5 per cent. per annum, compound intereſt?

```
           1.157625
                225
           --------
            5788125
           2315250
          2315250
          ---------        £.  s. d.
Anſwer    260.465625   or  260  9  3 ¾
```

N. B. If it is required for more years than what are in the table, take any two or more numbers as make the number of years you want, and multiply the ſums belonging to each number in the table together, which gives the ſum for that number of years; and that ſum multiply'd by the principal gives a direct anſwer.

A Table ſhewing the amount of 1l. for days, at 5 per cent: per ann. compound intereſt.

Days.	£.
1	1.0001337
2	1.0002674
3	1.0004011
4	1.0005348
5	1.0006686
6	1.0008023
7	1.0009361
8	1.0010699
9	1.0012038
10	1.0013376
20	1.0026770
30	1.0040182
40	1.0053612
50	1.0067060
60	1.0080525
70	1.0094009
80	1.0107511
90	1.0121031
100	1.0134569
110	1.0148125
120	1.0161699
130	1.0175291
140	1.0188902
150	1.0202531
160	1.0216178
170	1.0229843
180	1.0243527
190	1.0257228
200	1.0270949
250	1.0339825
300	1.0409164
365	1.0499999
viz.	1.05

Example. What will be the amount of 170l. 10s. in 25 years, at 5 per cent. per annum, compound intereſt?

```
         2.653298
         1.276282
      -------------
          5306596
        21226384
        5306596
      15919788
     18573086
     5306596
    2653298
      -------------
    3.386356|478036
          170.5
      -------------
     16931780
   23704492
   3386356
      -------------
Anſ. 577.3736980
   or £. 577  7  5 ½
```

Suppoſe I wanted to know what would be the amount of 1l. for any number of days not in the table, as ſuppoſe for 194 days; then look in the table for the number againſt 190, and alſo againſt 4, and multiply thoſe two numbers together, the product is the amount of 1 l. for 194 days; proceed in the ſame manner for any other number of days.

And ſuppoſe I would know the amount of 1l. for 5 years and 194 days, proceed thus; againſt 5 years is 1.276282, and the amount of 1l. for 194 days is 1.026271; the product is the amount of 1l. in 5 years and 194 days equal to 1.30981, &c.

N. B. This table of the amount of 1l. for days calculated at compound intereſt, ought to be made uſe of inſtead of the vulgar method in uſing ſimple intereſt for days, as may be ſeen by what follows; and as it correſponds with what has been ſaid before on ſimple intereſt.

The ſimple intereſt of one pound is a ſeries of terms in arithmetic progreſſion continually increaſing, but compound intereſt is a ſeries of terms in geometrical progreſſion increaſing; and conſequently the amount of any ſum of money reckoned at ſimple intereſt will be very different from that reckon'd at compound intereſt whether it be for more or leſs than an year: See Examples.

EXAMPLE FOR YEARS.

Thus 100 l. at 5 per cent. for 6 years ſimple intereſt amounts to 130l. and 100 at 5 per cent. for 6 years compound intereſt gives 134.0096l.

Example for Days.

100 l. at 5 per cent. for 80 days ſimple intereſt amounts to 101l. 1s. 11d. and 100 at 5 per cent. for 80 days compound intereſt gives 101l. 1s. 6d. for the amount; the difference is 5d.

In order to illuſtrate this ſubject a little further, let us conſider that if a perſon pays intereſt at any leſs time than an year, he ought to have intereſt for the money paid correſponding to the time it is paid before it becomes due; for inſtance, if he pays the intereſt half yearly of 100l. at 5 per cent. in this caſe ſuch a ſum muſt be paid as if he ſhould put this intereſt money of 100l. for 6 months to uſe it wou'd gain him ſo much money, which ſums added to the intereſt money of 100l. for the other 6 months, together with the principal ſhould amount to 105l. at the year's end; and conſequently it muſt be reckoned at compound intereſt for any time leſs than an year, as well as for any time greater than an year. Now let us ſuppoſe this example to be reckon'd at ſimple intereſt, and then we ſhall ſee by the conſequence whether it amounts juſt to 105l. at the end of the year.

The intereſt of 100 l. for one day at 5 per cent. ſimple intereſt is .01369863, which multiplied by 182½ the days in half a year gives the intereſt of 100 l. for ½ an year,

	£.	s.	d.
equal to —— ——	2	10	0
And the 2l. 10s. being put to intereſt for the other ½ year will gain	0	1	3
Now the 100l. goes on again for the other half year and conſequently gains —— ——	2	10	0
To which add the principal money	100	0	0
And the ſum is —— ——	105	1	3

Inſtead of 105, which is 15d. too much, and conſequently muſt not be calculated at ſimple intereſt.

QUESTION. If a farthing was put to intereſt at 5 per cent. in the firſt year of the Chriſtian Æra, what would it amount to if it continued out at compound intereſt till the year of our LORD * 1765?

The amount of 1l. for 100 years is 131.50126; which multiplied into itſelf gives 17292.58138 for 200 years; that ſum multiply'd by itſelf gives 299033370.784 for 400 years; and that multiply'd by itſelf gives 89420956842441224.8 for 800 years; which ſum multiply'd by itſelf gives 7996107522617726100561034842524135 for 1600 years. Now the amount of 100 years is

---

* It is commonly ſuppoſed that the date of the year is the number of years ſince the birth of Chriſt, and indeed it ought to be ſo; but it is eaſy to prove that we reckon 4 years too little; for the vulgar Æra of Chriſt's birth was fixed by Dionyſius Exiguus a Roman abbot, to the end of the year of the Julian period 4713, which ought to have been fixed to the end of the 4709th year of the Julian period; and which is thus demonſtrated.

Chriſt was undoubtedly born before the death of Herod the great; but according to Joſephus (B. 17. ch. 8.) there was an Eclipſe of the Moon in the time of Herod's laſt illneſs; which very eclipſe our aſtronomical tables ſhew to have been in the year of the Julian period 4710 Mar. 13 d. 3 h. 21 m. after midnight at Jeruſalem. Now there is reaſon to ſuppoſe that Chriſt was about an year old when Herod gave command for the children to be ſlain from two years old and under; for according to this command Herod concluded that he was more than one year old yet leſs than two; and after this command he was in Egypt,

is 131.50126, which multiplied by 23.839901 the amount of 65 years, the product will be
I 3134.977

Egypt, and all this was before the death of Herod; however ſuppoſing inſtead of an year old we ſhall ſuppoſe him only a few months old at Herod's death, even then the lateſt time in which we can poſſibly fix the Æra of his birth muſt be about the end of the 4709th year of the Julian period.

Again, if a man will give himſelf the trouble, he may find the age of Chriſt when he was crucified, by this means, *viz.* if he will find the times of all the paſſover full moons from the 20th to the 40th year of Chriſt, after the Jewiſh manner by adding 14 days to the time when the preceding new moon was firſt viſible at Jeruſalem; he will find that the only paſſover full moon which falls on a Friday in all that time is in the year of the Julian period 4746, and on the 3d day of April. Now the difference between 4709 and 4746 is 37 years for the age of Chriſt when he was crucified, tho' generally ſuppoſed to be only 33 years old. Now this agrees with what Phlegon an heatheniſh writer and others tell us, "that in the laſt year "of the 202d olympiad there was the moſt extraordinary "eclipſe of the ſun that ever was known, and that it was "night at the 6th hour of the day." Which agrees exactly with the time that the darkneſs at the crucifixion began, and therefore muſt have been the very ſame darkneſs, but miſtaken by Phlegon for a natural eclipſe of the ſun, which was impoſſible becauſe the moon was then in the full; ſo that this darkneſs muſt have been quite out of the common courſe of nature. And as all theſe accounts agree, ſo conſequently the date of the year 1765 ſhould be 1769; which if this had been conſidered in the queſtion, and taken into the account, we ſhould have had above 5627000000000000000000000000000000 £. to be added to the amount of the 1765 years, on account of the 4 years extraordinary; and this is above 200,000 globes of fine gold each as big as the Earth that it amounts to in theſe laſt 4 years.

3134.977 for the 165 years; then that ſum multiply'd into the amount of 1600 years gives 25067613172933551117558531327511785169.9 for the amount of 1 l. for 1765 years. Laſtly, reduce 1 farthing to the decimal of a pound, viz. 960)1.0000000(.0010416; ——then multiply the amount of 1 pound for 1765 years by .0010416, and the product will be £. 26097056487235355616786268347361602.4, anſwering to the amount of one farthing in 1765 years at 5 per cent. per ann. compound intereſt.

CASE II. *To find the amount of any annuity, penſion, &c.*

RULE. Multiply the reſpective number in the table by the annuity, ſubtract the annuity from the product, and divide by the rate; the quotient is the amount, &c.

EXAMPLE. What will an annuity of 50 l. per annum amount to in 4 years at 5 per cent. per annum, compound intereſt?

```
     1.215506
           50
     --------
    60.775300
    50
     --------
.05)10.775300
     --------          £.   s. d.
Anſwer  215.5060 or 215 10 1 ¼
```

CASE III. *To find the preſent worth of annuities, penſions, &c.*

RULE. Divide the annuity by the tabular number, and ſubtract the quotient from the annuity, divide the remainder by the rate, the quotient is the preſent worth, &c.

EXAMPLE. What is the preſent worth of an annuity of 30l. per annum, to continue 7 years at 5 per cent. per annum, compound intereſt?

30
1.4071)30.0000000 (21.320446
28142
.05) 8.679554
18580 £. s. d.
14071 173.59108 or 173 11 10
Anſwer.
45090
42213
28770
28142
62800
56284
65160
56284
88760
84426
4334

CASE IV. *To find the preſent worth of any ſum.*

RULE. Divide the ſum by the tabular number and the quotient is the preſent worth.

EXAMPLE. What is the preſent worth of 420l. 12s. payable 4 years hence, at the rate of 5 per cent. per annum, compound intereſt?

1.215506)420.600000(346.0287
3646518

5594820
4862024

7327960
7293036

3492400
2431012

10613880
9724048

8898320
8508542

389778

£. s. d.
Anſ. 346 0 6 ¾

CASE V. *To find the annuity that any ſum will purchaſe.*

RULE. Add one to the number of years, take the number againſt it in the table, ſubtract the tabular number for the given years therefrom, the remainder is a dividend; then ſubtract 1 from the tabular number and divide the dividend by the remainder, the quotient multiply'd by the principal is the anſwer.

EXAMPLE. What annuity will 60l. purchaſe for 5 years continuance, at the rate of 5 per cent. per annum, compound intereſt?

```
          1.340096
          1.276282
         ----------
.276282).0638140000 ( .230974
          552564              60
          ------      ----------
           855760     13.858440
           828846
          -------
           2691400              £.  s.  d.
           2486538      Anſwer  13  17  2
          -------
            2048620
            1933974
           -------
             1146460
             1105128
            -------
               41332
```

Question. Suppoſe the national debt be 130 millions; what annuity will be ſufficient to pay it off in 30 years at 4 per cent. per annum, compound intereſt?

Amount of 1 l. for 30 years at 4 per cent. is —

```
                 3·2433975
                      1·04
             -------------
                 129735900
               32433975
             -------------
               3·373133400
               3·2433975
             -------------
2.2433975) ·129735900  (  ,05783
             112169875          130000000
             ---------     ------------------
              175660250      173490000000
              157037825      5783
             ----------    ------------------
               186224250 £·7517900,00000
               179471800        Annuity.
             ------------
                 67524500
                 67301925
             ------------
                   222575
```

So that if the national debt is 130 millions, and the ſum paid be 7517900 l. every year, that will clear the whole debt in 30 years.

Or

OF PURCHASING FREEHOLD ESTATES.

To find the preſent worth of any annual rent, to continue for ever, commonly called Fee Simple.

RULE.

DIVIDE the propoſed rent by the intereſt of 1l. for 1 year, and the quotient is the value of the eſtate.

EXAMPLES.

What is an eſtate of 200l. per annum, to continue for ever, worth in ready money, allowing the purchaſer 5 per cent. per annum, compound intereſt?

.05)200.00

£. 4000 Anſwer.

A has the poſſeſſion of an eſtate of 130l. per annum, to continue 20 years; B has the reverſion of the ſame, from that time for ever; what muſt A give B, if he wou'd purchaſe his reverſion? And what muſt B give A, if he would buy his poſſeſſion, accounting 5 per cent. per annum, compound intereſt, in each caſe?

See the work in the next page.

```
2.653298)130.000000(  130
         10613192      48.9956
         --------     ---------
         23868080  .05)81.0044
         21226384     ---------
         --------      1620.088
          26416960    A's poſſeſſion.
          23879682
          --------
           25372780
           23879682
           --------
            14930980
            13266490
            --------
             16644900
             15919788
             --------
               725112

              £.
         .05)130.00
             ------   s. d.
              2600    0  0
              1620    1  9
             -------------
B's reverſion  979   18  3
             -------------
```

A agrees with B for an annuity of 600l. per annum, to continue 25 years, to give him the preſent worth of it at 6 per cent. per annum, compound intereſt; but not having money enough by him, offers to make over to him a freehold eſtate of 12l. per annum at the ſame intereſt; what money beſide will pay his purchaſe?

```
                                600
4.29187)600.00000  ( 139.7992
        429187       ————————        .06)12.00
        ———————  .06)460.2008         ————
        1708130      ————————          200
        1287561       7670.013
        ———————        200
         4205690      ————————
         3862683      7470.013    £.   s. d.
         ———————        Answer 7470  0  3
          3430070     ——————————————————————
          3004309
          ———————
           4257610
           3862683
           ———————
            3949270
            3862683
            ———————
             865870
             858374
             ——————
               7496
```

Note. If a freehold eſtate be purchaſed, and 2 per cent. allowed, it is 50 years purchaſe, &c. as in the following table.

Rates.	Years purchaſe.
2	50.
2 ½	40.
3	33.3
3 ½	28.5714
4	25.
4 ½	22.2
5	20.
5 ½	18.18
6.	16.6
6 ½	15.3846
7	14.2857
7 ½	13.3
8	12.5
8 ½	11.7647
9	11.1
9 ½	10.5263
10	10.

THE VALUATION OF LEASES OR ANNUITIES ON LIVES.

AS this is a neceſſary ſubject, we ought not to paſs over it in this place; and though it has been handled by many learned gentlemen, yet the theorems and rules that they have given are ſo tedious and perplexed that they can only be of uſe to perſons with an uncommon ſhare of learning; this evil we ſhall endeavour to avoid, by giving ſuch rules, that any perſon with a moderate ſhare of learning may know the value of any annuity or leaſe for lives in a very ſhort time.

And we muſt firſt obſerve, that before the value of any annuity on a life of any propoſed age can be aſcertained, it will be neceſſary to conſider how we are to eſtimate the probability of the continuance of a life for any given time; and it is evident this can be done no other way but from obſervations made from the bills of mortality for a ſeries of years together, and ſuch as are proper for the country where theſe obſervations are to be made.—In London theſe bills are drawn up by the company of pariſh clerks, and contain an account of the numbers, ages, diſeaſes, &c. of ſuch as die within the bills of mortality, that is, in London, Weſtminſter, and 10 miles round. But the great diſparity between the births and burials in London is owing to this, that the diſſenters of all ſorts baptize their children without ſending an account of them to the pariſh clerks, ſo that little dependance is to be had on theſe with regard to calculating annuities for life. Dr. Hally's table founded on the Breſlaw bills of mortality is of much more authority; but as theſe bills of mortality will not ſuit England *, we ſhall firſt exhibit a table of the number of perſons who have died in the ſpace of 20 years, and then give a table of the probabilities of life from Mr. Simpſon.

			Mean.
Between	0 and 2	194645	9732
	2 and 5	45984	2299
	5 and 10	19037	952
	10 and 20	16575	829
	20 and 30	40858	2043
	30 and 40	49709	2485
	40 and 50	50945	2547
	50 and 60	41311	2066
	60 and 70	32486	1625
	70 and 80	23611	1181
	80 and 90	12198	610
	90 and 100	2083	104
	100 and upwards.	181	9

The

---

* Although Sileſia (of which Breſlaw is the capital) is much in the ſame latitude with England, and conſequently in the ſame climate, yet it is obſerved that the ſtate of the air and weather is much more regular at Breſlaw than in England, for the frequent and ſudden change of the air from hot to cold and from cold to hot in England, eſpecially in the ſpring, beſides that moiſt damp air that we have here in winter, makes the air much more unwholeſom than at Breſlaw. Again, an Engliſhman generally makes more free with his conſtitution than a Sileſian; ſo that the tables of the value of an annuity for lives being calculated by the Breſlaw bills of mortality would give numbers too large for a purchaſe to be made in England.

The following is Mr. SIMPSON's table of the probabilities of life.

	0	1	2	3	4	5	6	7	8	9	
Born	1280	870	700	635	600	580	564	551	541	532	Perſons living.
10	524	517	510	504	498	492	486	480	474	468	
20	462	455	448	441	434	426	418	410	402	394	
30	385	376	367	358	349	340	331	322	313	304	
40	294	284	274	264	255	246	237	238	220	212	
50	204	196	188	180	172	165	158	151	144	137	
60	130	123	117	111	105	99	93	87	81	75	
70	69	64	59	54	49	45	41	38	35	32	
80	29	26	23	20	17	14	12	10	8	6	

N.B. Mr. *Simpſon*'s table of the probabilities of life was conſtructed from the London bills of mortality, but by the allowances made it differs from all others in this, that it is better adapted for country towns and villages than for large cities, as London, Paris, Pekin, &c. becauſe it has been often obſerved that people in large and populous cities are more ſubject to mortality than in country places; and therefore on this account Mr. *Simpſon*'s table ought to have the preference of all others for general uſe.

By the help of this table we find what probability there is for a man of a certain age, 30 for example, ſhall live 1 year; thus againſt 31 is 376 and againſt 30 is 385, the meaning of which is that out of 385 perſons living at 30, there is only 376 living at 31: Thus the probability of a perſon of 30 years of age ſhall live one year is meaſured by this fraction $\frac{376}{385}$, that is, he has the odds of 385 to 9, or nearly 43 to 1 that he does not die in one year.

So the odds that a man of 34 ſhall live 7 years is found by ſubtracting 284 in the table from 349; hence the odds is 284 to 65, or 4 $\frac{1}{3}$ to 1, that a man of 34 lives 7 years; and the like for any other number of years.

Hence it appears, that the value of inſurance upon lives ought to be regulated, there being a great difference between inſuring the life of a man of 20, and that of another of 50 years of age; ſince 'tis 65 to 1 that the man of 20 dies not in an year, and but 24 $\frac{1}{2}$ to 1 for a man of 50 years of age.

The moſt general method of valuing annuities on lives is from Mr. de Moivre; *viz.* Take the value of an annuity certain for ſo many years as are denoted by the complement of life (which is what the given life wants of 86;) multiply this value by the rate of intereſt, and divide the product by the complement of life; ſubtract the quotient from 1, divide the remainder by the intereſt of 1 pound, and this laſt quotient will expreſs the value of an annuity for any age given.

But I think we can get an eaſier rule than this of Mr. de Moivre's from the following conſideration. Suppoſe a perſon A was to receive

1 l. upon this condition, that another, B, 20 years of age ſhall live 1 year; quere the value of A's expectation?

Againſt the ages of 20 and 21 are 462 and 455; now the preſent value of 1 l. due at the end of one year at 5 per cent. is .95238, which multiplied by this fraction $\frac{455}{462}$ (that is, multiplying by the numerator and dividing by the denominator) gives .938 of a pound for the true value required.

In like manner the probability of a perſon of 20 years of age has of living 2 years is $\frac{448}{462}$, and the preſent worth of 1 l. due at the end of 2 years at 5 per cent. is .907; therefore multiply .907 by $\frac{448}{462}$, and it gives .8795 of a pound for the value of A's expectation to receive 1 l. at the end of the 22d year of B's life. And thus you may proceed for all the other years of his life to the extremity of age; and the ſum of all theſe (from 20 and upwards) being found, and added together, will amount to 13 l. very nearly.

It is demonſtrated by the writers on the value of lives, that the ſum of the values of 1 l. for the ſucceſſive years of a perſon's life will be found by dividing 1 by the numbers in the table of the amount of 1 l. which numbers being added together will give the value of that life which is equal to the number of theſe numbers ſo added, if there was no contingency in the caſe; but as we muſt allow for that, there ariſes a long and tedious algebraic operation, which is too long to be inſerted here; but the rule thence ariſing is this;

To the value of the given life add one year's purchaſe, diſcount that ſum, and multiply it by the probability of a life of one year younger than the given life, this laſt product will be the value of an annuity upon this life.

For example, the life of 20 being 13 l. this increaſed by one year's purchaſe is 14, which diſcounted at 5 per cent. is 13.3, this multiplied by the probability of a life of 19, gives 13.16 for the required value of a life of 19 years of age: And in this manner are the numbers in the following table computed, which ſhews the value of an annuity on any ſingle life from 1 to 80, at the rate of 3, 4, 5 and 6 per cent. compound intereſt.

N. B. The numbers in the following table, and alſo the anſwers to the queſtions on annuities on lives ſhew the pounds that will purchaſe one pound annuity on ſuch lives; or they ſhew the number of years purchaſe that any life or lives (contained in the table) are worth; therefore, multiply the numbers in the table, or the anſwers to the queſtions on annuities on lives by the annuity, and the product ſhews the value of that annuity.

The following is a table ſhewing the preſent value of an annuity of 1 l. (or, which is the ſame, the number of years value which ſuch annuity is worth) for a life of any age under 81, at 3, 4, 5, and 6 per cent. compound intereſt.

Ages.

Ages	3	4	5	6
9&10	19.0	16.4	14.3	12.7
8. 11	19.0	16.4	14.3	12.6
7. 12	18.9	16.3	14.2	12.5
6. 13	18.7	16.2	14.1	12.4
14	18.5	16.0	14.0	12.4
5. 15	18.3	15.8	13.9	12.3
16	18.1	15.6	13.7	12.2
17	17.9	15.4	13.5	12.0
4. 18	17.6	15.2	13.4	11.9
19	17.4	15.0	13.2	11.8
20	17.2	14.8	13.0	11.6
21	17.0	14.7	12.9	11.5
3. 22	16.8	14.5	12.7	11.3
23	16.5	14.3	12.6	11.2
24	16.3	14.1	12.4	11.1
25	16.1	14.0	12.3	11.0
26	15.9	13.8	12.1	10.8
2. 27	15.6	13.6	12.0	10.7
28	15.4	13.4	11.8	10.6
29	15.2	13.2	11.7	10.5
30	15.0	13.1	11.6	10.4
31	14.8	12.9	11.4	10.2
32	14.6	12.7	11.3	10.1
33	14.4	12.6	11.2	10.0
34	14.2	12.4	11.0	9.9
1. 35	14.1	12.3	10.9	9.8
36	13.9	12.1	10.8	9.6
37	13.7	11.9	10.6	9.5
38	13.5	11.8	10.5	9.3
39	13.3	11.6	10.4	9.2
40	13.2	11.5	10.3	9.1
41	13.0	11.4	10.2	9.0
42	12.8	11.2	10.1	8.9
43	12.6	11.1	10.0	8.8
44	12.5	11.0	9.9	8.7
45	12.3	10.8	9.8	8.6
46	12.1	10.7	9.7	8.6
47	11.9	10.5	9.5	8.5
48	11.8	10.4	9.4	8.4
49	11.6	10.2	9.3	8.4
50	11.4	10.1	9.2	8.3
51	11.2	9.9	9.0	8.2
52	11.0	9.8	8.9	8.1
53	10.7	9.6	8.8	8.0
54	10.5	9.4	8.6	7.8
55	10.3	9.3	8.5	7.7
56	10.1	9.1	8.4	7.6
57	9.9	8.9	8.2	7.5
58	9.6	8.7	8.1	7.4
59	9.4	8.6	8.0	7.3
60	9.2	8.4	7.9	7.2
61	8.9	8.2	7.7	7.1
62	8.7	8.1	7.6	7.0
63	8.5	7.9	7.4	6.9
64	8.3	7.7	7.3	6.7
65	8.0	7.5	7.1	6.6
66	7.8	7.3	6.9	6.5
67	7.6	7.1	6.7	6.3
68	7.4	6.9	6.6	6.2
69	7.1	6.7	6.4	6.1
70	6.9	6.5	6.2	5.9
71	6.7	6.3	6.0	5.7
72	6.5	6.1	5.8	5.6
73	6.2	5.9	5.6	5.4
74	5.9	5.6	5.4	5.3
75	5.6	5.4	5.2	5.1
76	5.3	5.2	5.0	4.9
77	5.0	4.9	4.7	4.6
78	4.7	4.6	4.4	4.3
79	4.3	4.2	4.0	3.9
80	3.9	3.8	3.6	3.5
81			3.	

We now proceed to compute the value of annuities on the longeſt lives of two lives, on the ſame principles as before; but we ſhall not give the inveſtigation of the theorems or rules, but only the rules themſelves, becauſe of the intricacy and difficulty attending it, this work being not deſign'd for Algebraiſts; and ſecondly, becauſe it would take up more room than we can poſſibly ſpare for this purpoſe.

CASE I. *If both the lives are equal.*

RULE. 1ſt. Divide the complement of a ſingle life by 6 times the amount of 1 l. for one year; 2d. ſubtract the quotient from the value of one of the lives; 3d. add half the remainder to the value of one life, and the ſum is the value of both lives.

QUEST. What is the value of the longeſt of two equal lives each of 48 years; allowing compound intereſt at 4 per cent. per annum?

Amount of 1l. for 1 year 1.04
86
6
Life 48
Complement 38
6.24)38.00(6.08
3744
5600
4992
608

Value of a life of 48 at 4 per cent. } 10.4
6.08
2)4.32
2.16
10.4
Anſwer 12.56

Case II. *If the lives are both unequal.*

Rule. 1ſt. Divide the complement of the oldeſt life by 6 times the amount of 1l. for one year; 2d. ſubtract the quotient from the value of the oldeſt life; 3d. multiply the remainder by the complement of the oldeſt divided by twice the complement of the youngeſt; 4th. add this product to the value of the youngeſt life, and the ſum will be the value of the longeſt of the two lives.

Quest. What is the value of an annuity of 1l. to continue during the life of the longeſt liver of two perſons, whoſe ages are 43 and 54; allowing 4 per cent. compound intereſt?

86 86
43 54
— —
43 32

1.04
6
———
6.24)32.00(5.128
3120
———
800
624
———
1760
1248
———
5120
4992
———
128

Value of the oldeſt life. 9.4
5.128
———
4.272
.372
———
8544
29904
12816
———
1.589184
Value of the youngeſt life. 11.1
———
12.689184 Anſ.

43
2
———
86)32.0(.372
258
———
620
602
———
180
172
———
8

To find the preſent value of an annuity for the longeſt of three lives; in this is four caſes.

1ſt. CASE. *When all three lives are equal.*

RULE. 1ſt. Divide the complement of a ſingle life by 6 times the amount of 1l. for one year; 2d. ſubtract the quotient from the value of a ſingle life; 3d. take ¼ of the remainder and add it to the value of a ſingle life, the ſum is the anſwer.

QUEST. Let the three perſons be each 54 years old, and compound intereſt be allow'd at 4 per cent. what is the preſent value of the longeſt liver?

```
 86                            1.04
 54                               6
 ——                            ————
 32                         6.24)32.00(5.128
                                 3120
                                 ————
Value of a ſingle life 9.4        800
                       5.128      624
                       —————      ———
                       4.272     1760
                           3     1248
                       —————     ————
                    4)12.816     5120
                       —————     4992
                       3.208     ————
                       9.4        128
                       —————
               Anſwer 12.608
```

2d. **Case.** *When the two youngeſt are equal.*

Rule. 1ſt. Divide the complement of one of the youngeſt lives by 6 times the amount of 1l. for one year; 2d. ſubtract the quotient from the value of one of the youngeſt lives, and take half the remainder and call it A; 3d. divide the complement of the eldeſt life by 6 times the amount of 1l. for one year; 4th. ſubtract this quotient from the value of the eldeſt life; 5th. multiply the remainder by the ſquare of the complement of the eldeſt life, and divide by 4 times the ſquare of the complement of one of the youngeſt lives; add this, and A, and the value of one of the youngeſt lives together, the ſum is the value required.

Quest. Let the two youngeſt be 34 and the eldeſt 54, what is the value of an annuity for the longeſt of the three lives, at 4 per cent.?

```
86                                  1.04
34      1.04                           6
——         6                        ————
52      ————                        6.24)52.00(8.3
——      6.24)32.00(5.128                 4992
86           3120                        ————
54           ————                         2080
——            800                         1872
32            624                         ————
——           ————                          208

              1760  Value of one of the }
              1248     youngeſt lives   } 12.4
              ————                          8.3
              5120                         ————
              4992                       ½)4.06
              ————                         ————
               128                       A 2.03
```

See the remainder of the work in the next page.

```
  32       52          9.4  Value of the eldeſt
  32       52          5.128      life.
 ----     ----        -------
  64      104          4.272
  96      260          1024
 ----     ----        -------
 1024     2704         17088
             4         8544
          -----       4272
          10816       -------
            10816)4374.528(  .404
                  43264      2.033
                  -------   12.400
                   48128    -------
                   43264    14.837 Anſw.
                  -------   ------------
                    4864
```

3d. CASE. *When the two eldeſt are equal.*

RULE. 1ſt. Divide the complement of the oldeſt ſingle life by 6 times the amount of 1 l. for one year; 2d. ſubtract the quotient from the value of the oldeſt ſingle life; 3d. multiply the remainder by 3 times the complement of the oldeſt ſingle life, and divide by 4 times the complement of the youngeſt life; 4th. add this laſt number to the value of the youngeſt ſingle life, the ſum is the value of the longeſt of the three lives.

QUEST. What is the value of an annuity at 4 per cent. for the longeſt of 3 lives, the youngeſt is 20, and the two oldeſt are each 34?

See the work in the next page.

86 1.04
34 6
52 6.24)52.00(8.3
4992
86 2080
20 1872
52 66 208
3 4
156 264 Value of a life of 34 is } 12.4
8.3
4.06
156
2440
20333
40666
264)634.40 (2.4
528 14.8
1064 17.2 Anſw.
1056
8

4th Case. *If the lives are all unequal.*

Rule. Put A the youngeſt, B the next, and C the oldeſt life; and let *a*, *b*, and *c*, be their complement of life to 86. Then

1ſt. Divide *b* by 6 times the amount of 1l. for one year, ſubtract the quotient from the value of B's life, and multiply the remainder by *b*.

2d. Divide *c* by 6 times the amount of 1 l. for one year, ſubtract the quotient from the value of C's life, and multiply the remainder by *c* ſquared, and divide by twice *b*.

3d. Add theſe 2 ſteps together, and divide the ſum by twice *a*, add this to the value of A's life, and the ſum will be the value of the annuity for the longeſt of the 3 unequal lives.

QUEST. Let the 3 unequal lives be A 20, B 34, and C 54; what is the value of the longeſt, at 4 per cent.?

1.04
6

6.24)52.00 (8.3
4992

2080
1872

208.

Value of B's life. 12.4
8.3

4.06
52

813
20333

211.46

86	86
34	54
b 52	*c* 32

See the remainder of the work in the next page.

```
  52
   2
 ----
 104
 -----
    32
    32
 -----
    64
   96
 -----
  1024
 -----
   86
   20
 -----
   66
    2
 -----
 132
```

```
6.24)32.00(5.128
     3120
     -----
       800
       624
      -----
       1760
       1248
       -----
        5120
        4992
        -----
         128
```

```
         9.4 value of C's life.
         5.128
         -----
         4.272
          1024
         -----
         17088
         8544
       4272
  --------- 211.46 } add
104)4374.528(42.0628
    416      ---------
   ---- 132)253.5295 ( 1.9206     14.8 value of A's
      214    132                  ----          life.
      208    ----                 16.7206 Anſwer.
      ----   1215
       652   1188
       624   -----
      ----    272
       288    264
       208    ----
      ----     895
        80     792
               ----
               103
```

EXTRACTION of the SQUARE ROOT.

TO extract the ſquare root of any number, is to find out ſuch a number, as being multiply'd into itſelf, the product ſhall be equal to the number given.

All the ſingle ſquare numbers, with their reſpective roots, are contain'd in the following table.

Roots,	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81

1ſt. You muſt point your given number, that is, make a point over the unit's place, another upon the hundred's, and ſo upon every ſecond figure throughout.

2dly. Then ſeek in the table for the greateſt ſquare number in the firſt point towards the left hand, placing the ſquare number under the firſt point, and the root thereof in the quotient, ſubtract the ſaid ſquare number from the firſt point, and to the remainder bring down the next point, and call that the reſolvend.

3dly. Then double the quotient, and place it for a diviſor on the left hand of the reſolvend; ſeek how often the diviſor is contained in the reſolvend, (reſerving always the unit's place) and put the anſwer in the quotient, and alſo on the right hand ſide of the diviſor; then multiply by the figure laſt put in the quotient, and ſubtract the product from the reſolvend, bring down the next point to the remainder and proceed as before.

Note. If the given number be a mix'd number, viz. conſiſting of a whole number and a decimal together, make the number of decimal places even, that is, 2, 4, 6, &c. ſo that there may a point fall upon the unit's place of the whole numbers: The ſame method muſt be taken in finding the fractional part of the root, by adding cyphers by pairs to the remainder, &c.

EXAMPLES.

Let 106929 be a number given, and the ſquare root thereof be required? As alſo the ſquare root of this number 226.8741?

```
 106929)327                226.8741(15.06
   9       Root.            1            Root.
 62)169                   25)126
    124                      125
 647)4529               3006) 18741
     4529                     18036
                                705
```

Requir'd the ſquare root of .1024?

```
   .1024(.32
    9
 62)124
    124
```

What's the ſquare root of 56078967.3?

· · · ·
56078967.3(7488.58
49 Anſwer.

144)707
576

1488)13189
11904

14968)128567
119744

149765)882330
748825

1497708)13350500
11981664

1368836

The various *Uſes* of the SQUARE ROOT.

Firſt, to find a geometrical mean proportional between any two numbers.

RULE. Multiply one by the other, and extract the ſquare root of the product for the anſwer.

EXAMPLE. What's the mean proportional between 36 and 84?

84
36

504
252

3024

3024(54.99
25 Anſwer.

104) 524
416

1089)10800
9801

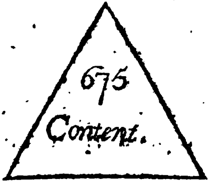
10989)99900
98901

999

PROBLEM. *To find the ſide of a ſquare that ſhall be equal to the content of any given ſuperficies.*

RULE. Extract the ſquare root of the given content for the ſide of a ſquare equal thereto.

EXAMPLE.



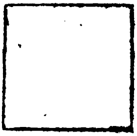
675(25.98 one ſide of the ſquare.
4

45)275
225

509)5000
4581

5188)41900
41504

396



Example. Admit 74685 ſoldiers being ordered into a ſquare battalia, that is, having as many in rank as file?

```
     ·  ·  ·
     74685(273
     4        Anſwer.
    ———
  47)346
     329
    ——————
 543)1785
     1629
    ——————
      156 odd men.
```

Note. If you would have a double battalia of men, extract the ſquare root of half the number given, and that gives the number to be plac'd in the file; then there muſt be twice as many in rank. Or if you would have a quadruple battalia, extract the root of ¼ of the given number, &c.

Problem. *Having any two ſides of a right angled plain triangle given, to find the other ſide.*

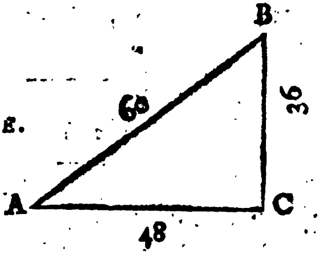
Theorem. In a right angled triangle, the ſquare of the hypothenuſe (or longeſt ſide) is equal to the ſquare of the baſe and perpendicular. There are three caſes belonging to the right angled triangle, and all depend upon this theorem.

1ſt. When the baſe and perpendicular are given; add the ſquare of the baſe to the ſquare of the perpendicular, and the ſquare root of the ſum is the hypothenuſe.

2d. When the baſe and hypothenuſe are given; from the ſquare of the hypothenuſe, ſubtract the ſquare of the baſe, and the ſquare root of the remainder is the perpendicular.

3d. When the hypothenuſe and perpendicular are given; from the ſquare of the hypothenuſe, ſubtract the ſquare of the perpendicular, and the ſquare root of the remainder is the baſe.

EXAMPLE.



CASE 1ſt. Given the baſe A C 48, and perpendicular B C 36, to find the hypothenuſe ?

```
     36                    48
     36                    48
    ----                  ----
    216                   384
   108                   192
   -----                 -----
   1296                  2304
                         1296
                         -----
                         3600(60
                         36       Anſwer.
                         -----
                           00
```

Case 2d. Given the baſe A C 48, and hypothenuſe A B 60, to find the perpendicular?

48	60	
48	60	
384	3600	
192	2304	
2304	1296(36	Anſwer.
	9	
	66)396	
	396	

Case 3d. Given the hypothenuſe A B 60, and perpendicular B C 36, to find the báſe?

36	60	
36	60	
216	3600	
108	1296	
1296	2304(48	Anſwer.
	16	
	88)704	
	704	

Pro-

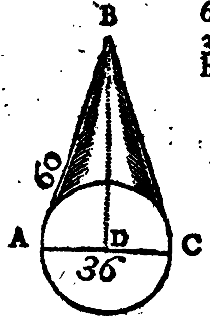
PROBLEM.

Having any two ſides of a cone, to find the other ſide.

By the help of the foregoing problem, you'll find, that to ſquare the ſlant height (which is the length of a line drawn from its vertical point to the circumference of its baſe) and from that ſquare ſubtract the ſquare of the ſemidiameter of its baſe, then the ſquare root of the remainder is the perpendicular height of the cone.

Note. The ſame rule ſerves for a pyramid.

EXAMPLE. Given the ſlant height A B 60, and baſe diameter A C 36, to find the perpendicular height B D ?



```
½)36        60
            60
 18        ----
 18        3600
----        324
 144       ----
18         3276(57.23 Anſ.
----       25
 324       ----
         107)776
             749
            ----------
           1142)2700
                2284
                ------
           11443)41600
                 34329
                 ------
                  7271
```

There is a tower whoſe height is 45 yards, and is ſurrounded by a moat 30 yards broad; of what height muſt a ſcaling-ladder be that will reach from the outſide of the moat to the top of tower?

Yards.

30
30
900

45
45
225
180
2025
900
2925

2925(54.08 Answer.
25
104)425
416
10808)90000
86464
3536

Two men ſet out together from one place; one went directly north 72 miles, and the other went directly weſt 96 miles; I would know how far they are aſunder?

72
72
144
504
5184

96
96
576
864
9216
5184

14400(120 miles. Answer.
1
22)44
44
00

Suppoſe a ſhip ſails 440 miles till her difference of latitude is 310 miles, I demand how far ſhe is departed from the meridian?

```
           440                   310
           440                   310
          -----                 -----
         17600                  3100
         1760                   930
        ------                 -----
        193600                 96100
         96100
       --------      Miles.
         97500(312.25 very near.
         9            Anſwer.
        ----
      61)75
         61
        -----
     622)1400
         1244
        ------
    6242)15600
         12484
        -------
   62445)311600
         312225
        --------
```

N. B. In this queſtion, the departure is the diſtance that the ſhip is to the eaſt or weſt of the meridian, and is the baſe line; the diſtance that the ſhip has failed is the hypothenuſe; and the difference of latitude is the perpendicular, and ſhews how much the ſhip is to the north or ſouth of the place it ſet ſail from.

A

A ladder 52 ½ feet long, being ſo placed in a ſtreet, that it reach a window 29 feet from the ground on one ſide, and by turning the ladder over (without removing the foot) it will touch a moulding 40 feet from the ground on the other ſide; how broad is the ſtreet?

```
     52.5          29                       40
     52.5          29                       40
   --------      ------                   ------
     2625          261       2756.25       1600
    1050           58        1600
   2625          ------     ---------
  ----------       841      1156.25(34.0
   2756.25                  9
    841                    ----------
                          64)256
  ------------                256
                             ---------
   1915.25(43.76                   25
   16
  ----
  83)315
     249
  ----
  867)6625            43.76
      6069            34.0
    ------           ------
  8746)55600   Anſ.  77.76  Feet.
       52476
      -------
       3124
```

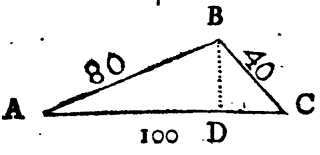
Alſo from the foregoing problem, there may be other examples very eaſily conſtructed to exerciſe the learner therein, as the three caſes all depend upon one theorem.

PROBLEM. *Having the three ſides of an oblique angled triangle given, to find the point in the baſe where the perpendicular of a right angled triangle ſhall fall.*

RULE. Square the two longeſt ſides, and from the ſum of theſe ſquares ſubtract the ſquare of the leſſer ſide, divide half the remainder by the longeſt ſide, and the quotient will give the anſwer.

EXAMPLE. Given the longeſt ſide A C 100, the next in length A B 80, and the leaſt ſide B C 40, to find the point D in the baſe where the perpendicular ſhall fall?

80	100	40
80	100	40
6400	10000	1600



10000
6400
16400
1600
½)14800
1|00)74|00
74 Answer.

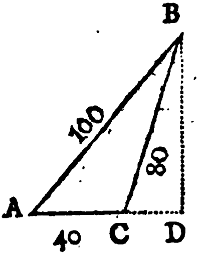
Note. To find the length of the perpendicular B D in the aforementioned triangle; from the ſquare of A B, ſubtract the ſquare of A D, and the ſquare root of the remainder is the perpendicular.

PROBLEM.

Having the ſides of an oblique angled triangle given, to find how far a perpendicular let fall from an acute angle, will fall without the triangle.

RULE. From the ſquare of the longeſt ſide ſubtract the ſum of the ſquares of the two leſſer, and divide half the remainder by the leaſt ſide, and the quotient will be the diſtance.

EXAMPLE. Given the longeſt ſide A B 100, the next in length B C 80, and the leaſt ſide A C 40, to find the point D where the perpendicular ſhall fall without the triangle?



80	40	100
80	40	100
6400	1600	10000
	6400	8000
	8000	$\frac{1}{2}$)2000
		4|0)100|0
		Anſwer 25

There

Example. There is a cask whoſe head diameter is 22 inches, bung diameter 25 inches, and the length 30 inches; what is the length of the diagonal line?

Rule. Add the ½ difference of the bung and head diameter to the head, and ſquare the ſum; add this ſquare to the ſquare of ½ the length, and the ſquare root of this ſum will be the diagonal required.

```
 25        22          15 half the length.
 22         1.5        15
----      -----       ----
½)3        23.5        75
----       23.5        15
   1.5    ------      ----
           1175        225
           705         552.25
           470        ------       Inches.
          ------       . . .
           552.25      777.25(27.9 very near.
                       4             Anſwer.
                      ------
                    47)377
                       329
                      ----
                   549)4825
                       4941
                      ----
```

Problem. *To find the diameter of a circle whoſe area ſhall be in a given proportion to that of a circle whoſe diameter is given.*

Rule. The given diameter { multiplied / divided } by the ſquare root of the intended { increaſe / decreaſe } will give the diameter of the circle.

What is the diameter of a circle whoſe area is 4 times as much as one of 11 inches diameter?

```
                        11
Square root of 4 is      2
                        --
             Anſwer     22  inches.
                        --
```

Admit a circle whoſe diameter is 63 inches; of what diameter muſt a circle be whoſe area will be 9 times leſs than that of the given diameter?

```
     Square root of 9 is 3)63
                           --
             Anſwer 21 inches.
```

Theorem. All like ſurfaces are to each other, as the ſquares of their like ſides, diameters, &c.

If the tranſverſe diameter of an ellipſis be 57, and conjugate diameter 41; what will be the diameter of a circle whoſe area will be equal to that of the oval?

```
  57
  41
----
  57
228
----
2337
```

See the remainder of the work in the next page.

2337(48.34 diameter requir'd.
16

88)737
704

963)3300
2889

9664)41100
38656

2444

What will be the diameter of a circle of an equal area to a parallelogram, whoſe length is 26.2 inches, and breadth 19.5 inches?

26.2
19.5

1310
2358
262

*.7854)510.9000(650.5
47124

39660
39270

39000
39270

650.50(25.5 in.
4 Anſwer.

45)250
225

504)2550
2520

30

---

* The reaſon of this number is demonſtrated in treating upon the circle in menſuration.

If a pipe of 3 inches diameter will fill a veſſel of water in an hour's time; of what diameter muſt a pipe be to fill the ſame in half an hour?

Inches.
3
3
M. —— M.
As 60 : 9 :: 30 (Inverſe proportion.)
9
——
3|0)54|0
——
Inches
18 (4.24
16 Anſwer.
——
82)200
164
——
844)3600
3376
——
224

If one fathom of cable whoſe circumference is 10 inches weighs 17lb. of what circumference muſt a cable be that one fathom thereof may weigh 10.88lb.?

Inches.
10
10
lb. —— lb.
As 17 : 100 :: 10.88
100
——
17) 1088.00 (64
102
——
68
68
——

64 (8 inches.
64 Anſwer.
——

In the midſt of a meadow well ſtor'd with graſs,
I took juſt an acre to tether my aſs;
How long muſt the cord be, that feeding all round,
He mayn't graze leſs nor more than his acre of ground?

```
   5.5
   5.5
  ----
  275
 275
 -----
 30.25 ſquare yards in
   160        a perch.
 ------
181500
3025
-------
4840 ſquare yds. in an acre.
```

```
.7854)4840.0000(6162.465
       47124
       -----
        12760
         7854
        -----
        49060
        47124
        -----
         19360
         15708
         -----
          36520
          31416
          -----
           51040
           47124
           -----
            39160
            39270
```

```
  .  .  .  .   ½
  6162.465(78.50 diameter.
  49          -----
 -----       39.25 yards.
148)1262      Anſwer.
    1184
    -----
1565)7846
     7825
     -----
      2150
```

A legal wincheſter buſhel is 18 ½ inches in diameter, and 8 inches deep; what muſt be the diameter of that buſhel whoſe depth is 9 inches?

```
.7854        2150 the ſolid inches in a buſhel.
    9
-----
7·0686)2150.0000(304.162
       212058
       ------
         294200
         282744
         ------
          114560
           70686
          ------
           438740
           424116
           ------
            146240
            141372
            ------
              4868
```

```
                 Inches.
          304.1620(17.44
          1        Anſw.
          --
       27)204
          189
          ---
      344)1516
          1376
          ----
     3484)14020
          13936
          -----
             84
```

What muſt be the diameter of a cylindrical veſſel of 3 feet deep, that ſhall hold twice as much as a veſſel of 28 inches deep, and 46 inches in diameter?

```
        12                  46              28
         3                  46               2
        --                 ---              --
        36                 276              56
                          184
                          ----
                          2116
                            56
                         -----
                         12696
                         10580
                        ------
                     36)118496(3291.5
                        108
           Inches.      ---
  3291.5(57.372         104
  25       Answ.         72
  --                    ---
107)791                 329
    749                 324
    ---                 ---
1143)4255                56
     3429                36
     ----               ---
11467)82655             200
      80269             180
      -----             ---
114742)238655            20
       229484
       ------
         9171
```

Suppoſe I drop a ſtone into a well, and it is 4 ſeconds before it ſtrikes the water at the bottom, how deep is the well?

N. B. Heavy bodies fall 16.13 feet the firſt ſecond, and the ſpace they paſs thro' is as the ſquare of the time of falling.

4	16.13
4	16
16	9678
	1613
Anſwer	258.08 feet.

Or thus.

The ſpace paſſed thro' by falling bodies in each ſingle ſecond is as the odd numbers 1, 3, 5, 7, 9, &c. therefore in the

Sec.		
1	it goes thro'	16.13
2	it goes thro' 3 times 16.13 equal to	48.39
3	it goes thro' 5 times 16.13 equal to	80.65
4	it goes thro' 7 times 16.13 equal to	112.91
	Same depth as before	258.08

But if you would know how deep that well is which is 4 ſeconds between the firſt deſcent of the ſtone and hearing the ſound it gives in ſtriking the water at the bottom, in ſuch caſe you muſt have recourſe to the following rule:

Square 1142 (the number of feet ſound flies per ſecond) and add 4 times the product of 16.13 and 1142 multiplied by the number of ſeconds it is falling, extract the ſquare root of the ſum, and ſubtract 1142 from the root, ſquare the remainder, and divide the ſquare by 4 times 16.13, and the quotient will be the true depth of the well.

```
    1142
    1142
 -------
    2284
   4568
  1142
 1142
 -------
 1304164
  294727.36
 -----------
 1598891.36(1264.4
 1             1142
 ----          ----
22)59          122.4
   44
   ----
246)1588
    1476
    ----
2524)11291
      10096
     -------
25284)119536
      101136
     -------
       18400
```

```
     16.13
     1142
    ------
     3226
    6452
   1613
  1613
 ---------
 18420.46
        4
 ---------
 73681.84
        4
 ---------
294727.36
```

See the remainder of the work in the next page.

```
                 122.4
                 122.4
                -------
                  4896
                 2448
      16.13     2448
          4    1224
     ------   -------   Feet.
      64.52 ) 14981.76 ( 232.2
              12904          Depth of the laſt
              -----                     well.
               20777
               19356
              ------
                14216
                12904
              -------
                 13120
                 12904
                -------
                   216
```

N. B. To count ſeconds, get a ſtring with a bullet at the end, and make it 39 ⅕ inches long, let it looſe and it will vibrate ſeconds; that is, each vibration will be a ſecond. Or if you have a uniform ſtaff or ſtick that is 58 ⅘ inches long, that will vibrate ſeconds alſo.

A

A new and expeditious method of extracting the Cube Root.

TO extract the cube root of any number, is to find ſuch a number, as being firſt multiplied by itſelf, and then into that product, produces the given number.

Rule. 1. Point every third figure of the given number beginning at units place; then find the neareſt cube to the firſt point; ſubtract, and bring the three next figures to the remainder for a reſolvend.

2d. Square the quotient and multiply it by 3 for a diviſor; find how many times it is contained in the reſolvend, rejecting units and tens, and put the anſwer in the quotient.

3d. Square this new figure, and put it on the right hand of the diviſor; but if the new figure ſhould be 1, 2, or 3, then put 01, or 04, or 09 to the right hand.

4th. Multiply the laſt figure in the quotient by 30, and multiply it by the former figures, add this product to the diviſor, and multiply the ſum by the laſt figure in the quotient, ſubtract that product from the reſolvend, bring down the next three figures and proceed as before.

A Table to find the neareſt cube to the firſt point.

Roots.	1	2	3	4	5	6	7	8	9
Cubes.	1	8	27	64	125	216	343	512	729

EXAMPLES.

Extract the cube root of 618470208(852
512 Answer.

New Divisor $\begin{array}{r}19225\\1200\\\hline 20425\end{array}$)106470
102125

New Divisor $\begin{array}{r}2167504\\5100\\\hline 2172604\end{array}$) 4345208
4345208

Extract the cube root of .009142439571(.2091
8 Answ.

New Divisor $\begin{array}{r}120081\\5400\\\hline 125481\end{array}$) 1142439
1129329

New divisor $\begin{array}{r}13104301\\6270\\\hline 13110571\end{array}$) 13110571
13110571

Extract the cube root 82153.680(43.47
64 Answer.

5169)18153
15507

559876)2646680
2239504

56597989)407176000
396185923

10990077

I have discontinued the work for the new divisors in this and the following examples.

N. B. In carrying on the decimal part of the quotient or root, you must add triple cyphers to the resolvend, as you do double in the square root. And in the proof, add or take in the remainder.

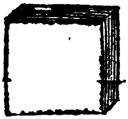
The

The various Uſes of the Cube Root.

THEOREM.

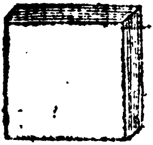
All like ſolids are to one another as the cube of their like ſides or diameters; and on the contrary, the cubes of their diameters are as their weights, &c.

There is a cube whoſe ſide is 6 feet; I demand the ſide of another cube whoſe ſolidity is double the former?



6 Feet.

```
        6
        6
      ———
       36
        6
      ———
      216
        2
      ———
      432 (7.55
      343
     ————
  15775)89000
        78875
       ——————
1698775)10125000
         8493875
        ————————
         1631125
```



Anſ. 7.55 Feet.

Suppoſe the content of a globe, or cylinder, or any given ſolid to be 1728 inches; what is the ſide of a cube equal thereto?

```
      ·  ·
      1728(12 inches.
      1   (Anſwer.
      ————
   364)728
       728
      ————
```

If an iron bullet that weighs 72 lb. is 8 inches in diameter; what will be the diameter of that bullet that weighs 9 lb.

```
              8
              8
             ——
             64
              8
             ——
   As 72 : 512 : : 9
              9
         ——————
      72)4608(64             64( 4 inches.
         432                 64  Anſwer.
         ————                ——
          288
          288
         ————
```

If a cannon ball of 4 inches diameter weighs 9 lb. what ſhall be the weight of another of 6 inches diameter of the ſame metal?

4 6
4 6
16 36
4 6
As 64 : 9 :: 216
9
lb.
64)1944 (30.375
192 Anſwer.
240
192
480
448
320
320

Problem. *Having the length, breadth, depth, &c. of any ſolid, or other veſſel, to find the like dimenſions of any other ſolid or veſſel in the ſame form as the other, to be in any proportion bigger or leſſer.*

Rule. As the ſolidity of the given body is to the cube of any dimenſion, (whether length, breadth, depth, diameter, circumference, &c.)

ſo is the ſolidity of the body requir'd to the cube of the like dimenſion, whoſe cube root will give the dimenſion itſelf which is at that operation ſought; and ſo with the reſt.

There is a ſhip of 60 tun burden, which meaſures 44 feet by the keel, middle 17.6 feet, and the depth of the hold 7.76 feet; it is required to make another ſhip of 360 tun burden of the ſame ſhape with the firſt?

Feet.
44
44
176
176
1936
44
7744
7744

T. T.
As 60 : 85184 : : 360
6

§
6|0)36|0 511104(80* feet.
6 512000 Keel.

§ This ſhort way of working the 1ſt. and 3d. term for a factor, anſwers the ſame purpoſe as if you multiplied by the 3d. term and divided by the 1ſt. and which method will be uſeful in many caſes.

* The true root ſhould be 79.9533, inſtead of 80, which is but about ½ an inch too long.

See the remainder of the work in the two following pages.

17.6
17.6

1056
1232
176

309.76
17.6

185856
216832
30976

As 60 : 5451.776 :: 360
6

32710.656 (31.98
27 Middle.

2791)5710
2791

296751)2919656
2670759

30604924)248897000
244839392

4057608

See the remainder of the work in the next page.

7.76
7.67
4656
5432
5432
60.2176
7.76
3613056
4215232
4215232

As 60 : 467.288576 :: 360
6

2803.731456 (14.10 Depth.
1
436)1803
1744
59221)59731
59221
510456

There is a conical tun which contains 620 gallons; the bottom diameter is 80, top diameter 71.2, and depth 30 inches; it is required to make another of the ſame form that will hold but 140 gallons?

See the work in the next page.

Inches.
80
80
6400
80

As 620 : 512000 : 140
140
20480000
512000

620)71680000(115612.9
620
968
620
3480
3100
3800
3720
800
620
1800
1240
5600
5580
20

115612.9(48.71 Inches. Bottom diam.
64
5824)51612
46592
701329)5020900
4909303
71165311)111597000
71165311
4043168

In the ſame manner is the top diameter found to be 43.35, and the depth 18.26 inches.

There is a caſk in form of the middle fruſtum of a ſpheroid, whoſe head diameter is 22 inches, bung diameter 25, length 30, and content 48.3 gallons; what is the head and bung diameters and length of another ſpheroidical caſk whoſe content is 80 gallons?

Inches.
22
22
44
44
484
22
968
968

G. G.
As 48.3 : 10648 : : 80
80

48.3)851840.0(17636
483
3688
3381
3074
2898
1760
1449
3110
2898
212

Inches.
17636.(26.02
8
1596)9636
9576
20295604)60000000
40591208
19408792

Anſwer 26.02 inches for the head diameter; and proceed in the ſame manner for the bung diameter and length.

How many 3 inch cubes can I have cut out of a cube whoſe ſide is 15 inches?

```
                 15
                 15
                ----
                 75
                15
                ----
                225
                 15
                -----
               1125
 3              225
 3             ----
---         27)3375(125 Anſwer.
 9             27
 3             ----
---             67
27              54
                ----
                135
                135
                ----
```

Or thus,

```
3)15
  ---
    5
    5
  ---
   25
    5
  ---
  125
  ---
```

Suppoſe a globe of 18 inches diameter; how many globes each of 4 inches diameter is equal thereto?

```
 4)18
 ------
   4.5
   4.5
  -----
   225
  180
  -----
  20.25
    4.5
  -----
  10125
  8100
  ------
Anſwer  91.125
  ------
```

A farmer borrow'd of his neighbour a cube of hay whoſe ſide was 8 feet, and pay'd back again 2 cubic pieces whoſe ſide was 4 feet each; quere, whether the lending farmer was fully paid?

```
   4                 8
   4                 8
  ---              -----
  16                64
   4                 8
  ---              -----
  64           128)512(4
   2               512
  ---              -----
 128
 ---
```

That is, 128 is but ¼ of 512, and conſequently the farmer paid but ¼ of what he borrow'd.

DUODECIMALS
OR
CROSS-MULTIPLICATION,

IS a method among workmen of conceiving every ſuperior place to be divided into twelves, and is of uſe only in caſting up the contents of their ſuperficial and ſolid works.

Moſt artificers take the dimenſions of their work in feet, inches, and parts, (viz. 12 parts;) others in feet, inches, and half quarters, or 8 parts; otherwiſe inſtead of 8 parts they take quarters, and reckon every quarter as 3 parts; ſo that theſe actually follow the duodecimal ſcale, altho' they ſeldom take notice of any other parts of an inch, than 3, 6, or 9, or give and take (as they call it) for the intermediate ones. And as in decimals, the places deſcend by tens from the units place to the right hand, ſo in duodecimals, the decreaſe is by twelves from the feet place to the right hand, and the ſame are noticed by feet, inches, parts, ſeconds, thirds, &c.

Obſerve if you multiply the length and breadth together, it gives the ſquare or ſuperficial content; and that product multiplied by the depth, gives the ſolidity. The eaſieſt method for multiplying duodecimally may be obtained by obſerving the following rule.

Firſt write down the feet, inches, &c. (to be multiplied together) in ſuch order, ſo that the denominations ſtand under thoſe of the ſame name, viz. ſeet under ſeet, inches under inches, &c. drawing a line underneath; then multiply each term in the multiplicand (beginning at the higheſt) by the feet in the multiplier, and write each reſult under its reſpective term: In the ſame manner multiply the multiplicand by the inches in the multiplier, and write the reſult of each term one place more to the right hand than before. Alſo obſerve the ſame method in multiplying by parts, ſetting the reſult of each term ſtill one place more to the right hand than in the laſt operation, &c. and the ſum of theſe reſults gives the product required.

N. B. Always when the multiplicand has feet, the firſt reſult of the multiplier (if you begin at the feet in the multiplicand) will fall directly under the multiplier and be of the ſame name; which method muſt be more feaſible, and not ſo liable to err as the common method of beginning to multiply the loweſt denomination of the multiplicand firſt, and carrying by twelves every time.

Alſo in the practical cuſtomary method, when the feet in the given dimenſions are many, that to multiply them by the leſſer denominations and take a twelfth of their product, will each time require ſome work to be done on a ſpare paper; but the method I have laid down will enable any one to perform the operations with more eaſe and leſs charge to the memory; for there will be no occaſion for any ſeparate work in this new method, except in adding the numbers up at laſt, and carrying the 12's to the next ſuperior place. (But ſhould the number of feet run very high, the operation may be better done by decimals than any way elſe.) Ex-

EXAMPLES.

If a board be 12 ½ feet long, and 15 inches broad, how many ſquare feet doth it contain?

```
       F.   I.
       12 — 6
        1 — 3
       ------
       12 — 6
            36 — 18
       -----------
Anſwer 15 — 7 — 6
```

If a board is 16 — 7 by 1 — 8, how many feet?

```
            F.   I.    F.  I.
            16 — 7  by 1 — 8
             1 — 8
            ------
            16 — 7
                 128 — 56
            -------------
Anſwer      27 — 7 — 8
```

Suppoſe a board to be 8 F. 6 I. and a half long, and broader at one end than the other, and the mean * we'll ſuppoſe to be 19 Inches and ¼; what the content?

```
         F.   I.   P.
         8 — 6 — 6
         1 — 7 — 3
         ---------
         8 — 6 — 6
             56 — 42 — 42
                  24 — 18 — 18
         ----------------------
Anſwer  13 — 8 — 5 — 1 — 6
```

---

* Thus in order to find a mean between the two ends of an oblong ſurface, ſuch as a board being one end

What does a marble ſlab come to, whoſe length is 5 feet 3 inches, and breadth 1 foot 9 inches and 4 parts, at 6s. 6d. per foot?

```
F.     I.     P.               In. |   |      s.  d.
1  —   9  —   4                 4  | ⅓ |      6   6
5  —   3                           |   |          9
----------------------             |   |  -----------
5  —  45  —  20                    |   |  2  18   6
       3  —  27  —  12             |   |      2   2
----------------------             |   |  -----------
9  —   4  —   0  —   0              £. 3      0   8 Anſ.
```

If an oak beam be 1 F. 6 I. by 1 F. 3 I. and 18 F. 3 I. long, how many ſolid feet are there?

```
                F.      I.
                1 ——— 6
                1 ——— 3
                ------------
                1 ——— 6
                      3 — 18
                ------------------
                1 —  10 ——— 6
Length         18 ——— 3
               -------------------
               18 — 180 — 108
                      3 —  30 — 18
               -------------------
Solidity       34 ——— 2 ——— 7 — 6   Anſwer.
```

---

broader than the other (having no regard to the thickneſs, or if you have it muſt be the ſame thro' the whole,) here you add both ends together and take half the ſum for a mean breadth; but this method will not ſerve for the mean of an unequal ſolid, ſuch as the fruſtum of a pyramid, &c. for then you muſt take the geometrical mean of the two ends, &c.

If a beam be 1 F. 5 I. 6 P. by 1 F. 1 I. 10 P. and 16 F. 3 I. 7 P. long, how many ſolid feet?

	F.	I.	P.			
	1	5	6			
	1	1	10			
	1	5	6			
		1	5	6		
			10	50	60	
	1	8	2	1	0	
Length	16	3	7			
	16	128	32	16		
		3	24	6	3	
			7	56	14	7
Solidity	27	4	9	7	5	7

Anſwer.

N. B. In ſome ſorts of work the anſwer will be requir'd in ſquare yards; then you are to divide the number of ſquare feet by 9 (the ſquare feet in a yard) and the quotient will be ſquare yards.

How many ſquare yards are there in a piece of work which meaſures 12 F. 7 I. 3 P. by 10 F. 5 I. 4 P.? And what will it come to at 3s. 3d. per yard?

See the work in the next page.

F. — I. — P.
12 — 7 — 3
10 — 5 — 4

120 — 70 — 30
60 — 35 — 15
48 — 28 — 12

9)131 — 7 — 8 — 8 — 0

Yds. — F. — I. — P. — S.
Content 14 — 5 — 7 — 8 — 8

d.
3 | ¼ | 14
3
42
3,6
2,0
2|0)4|7 6
£. 2 7 6 Coſt.

F. | | s. d.
3 | ⅓ | 3 3
1 | ⅓ | 1 1
1 | - | 4¼
4¼
2½ for the odd inches,&c.
2 0

Here you ſee when the number of feet runs pretty high, that the only difficulty is in adding up the numbers at laſt; and then the rule may be facilitated by having a table of the 12's contained in a pretty large number, by heart. Beſides, this as well as all other rules of the like nature, may be aided by the rules of practice, &c. And where the dimenſions contain but a few feet, you'll find this new method the moſt certain and moſt agreeable of any yet practiſed, except you take the dimenſions by decimals.

PLANI-

PLANIMETRY,

OR

MENSURATION OF SUPERFICIES.

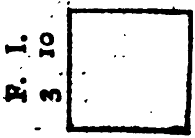
SUperficial figures are all ſuch as have only length and breadth, not having any commenſurable thickneſs.

PROBLEM I.

To find the area of a geometrical Square, or rectilineal Parallelogram.

RULE. Multiply the baſe by the perpendicular altitude (viz. the length by the breadth,) and the product will be the area or ſuperficial content.

EXAMPLE. Let the ſide of a ſquare be 3 feet 10 inches; what's the content in feet, inches, &c.?



F. I.
3 — 10
3 — 10
9 — 30
30 — 100
Anſwer 14 — 8 — 4

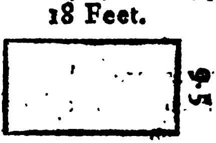
EXAMPLE. Suppoſe a ſquare in form of a parallelogram, 18 feet long, and 9.5 feet broad, what's the content?

By decimals.

Feet.
18
9.5

90
162

Facit 171.0



.5
12

6.0

By croſs-multiplication.

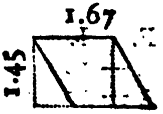
F. I.
18 — 0
9 — 6

162 — 0
108 — 0

Facit 171 — 0 — 0

The ſame rule holds for a rhombus or rhomboides; ſince 'tis evident from the two next figures that all parallelograms upon the ſame baſes and altitude are equal.

A Rhombus.

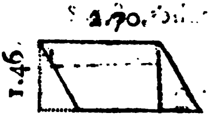


1.67
1.45

835
668
167

Area 2.4215

A Rhomboides.



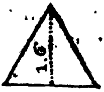
2.70
1.46

1620
1080
270

Area 3.9420

From theſe figures it will not be difficult to conceive, that the area of every triangle is the $\frac{1}{2}$ of the parallelogram that circumſcribes it; therefore the rule for obtaining the area of any triangle, is to multiply the baſe by the perpendicular altitude, and take the $\frac{1}{2}$ of that product for the area: Or if you multiply the baſe by half the perpendicular; or multiply half the baſe by the whole perpendicular; and any of theſe three ways will give the content.

An equilateral Triangle.



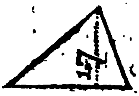
1.9
1.6

114
19

$\frac{1}{2}$)3.04

Area 1.52.

An acute Triangle.



$\frac{1}{2}$)1.70

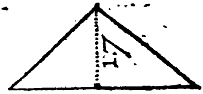
.85

2.45
.85

1225
1960

2.0825 Area.

An obtuſe Triangle.



$\frac{1}{2}$)1.70

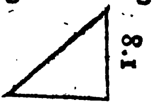
.85

3.92
.85

1960
3136

Area 3.3320

A right angled Triangle.



$\frac{1}{2}$)1.8

.9

2.03
.9

1.827
Area.

Obſerve, any angle greater than a right angle is obtuſe, and that which is leſs is acute.

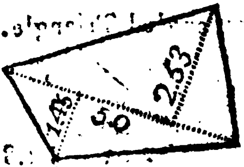
From the foregoing problem and corollaries likewiſe flow the following rule for finding the area of a trapezium and all irregular polygons.

To find the area of a Trapezium.

Firſt divide it into two triangles by a diagonal line drawn from the two moſt acute angles, and let fall two perpendiculars upon that diagonal line from the two remaining angles; then multiply half of that diagonal by the ſum of the two perpendiculars, and the product will be the area required.

Or if you add the two perpendiculars together, and take half the ſum and multiply it into the diagonal, that will give the content; or you may find the areas of the two triangles, and add thoſe two areas together for the content.

A Trapezium.



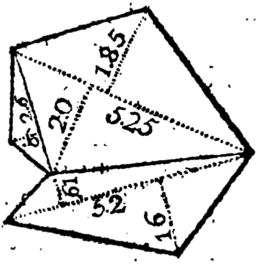
2.53
1.45
3.98
2.5
1990
796
Content 9.950

2)5.0
2.5

To find the content of any irregular figure, with more ſides than a trapezium.

Divide it into trapeziums or triangles or what elſe it will bear; ſo find the different areas ſeverally, and add them together for the content.

For example; give up the content of the following irregular figure.



1.85
2.0
½)3.85
1.925
5.25
9625
3850
9625
10.10625
5.746
.78
Area 16.63225

.61
1.6
½)2.21
1.105
5.2
2210
5525
5.7460

½)2.6
1.3
.6
.78

PROBLEM

PROBLEM II.

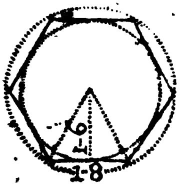
To find the area of any regular Polygon.

A regular polygon hath all its ſides equal, and is ſuch as can be inſcribed in a circle, ſo that all the angular points of it may be in the periphery or circumference of that circle which conſtitutes it. There are innumerable kinds, but the moſt common ones are theſe,

viz. a {
Pentagon of 5 ſides,
Hexagon of 6 ſides,
Heptagon of 7 ſides,
Octagon of 8 ſides;

All which or any other may be meaſured by the following rule: Multiply half the ſum of the ſides by the radius of the inſcribed circle; or multiply the ſum of the ſides by half the radius, and either of the products is the area.

EXAMPLE. Find the content of the annext hexagon in feet and inches.



F. I.
1 — 8
3
5 — 0
1 — 6
5 — 0
30 — 0
Content 7 — 6

PROBLEM III.

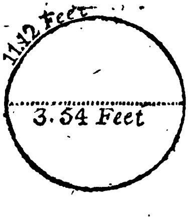
To find the area of any Circle.

A circle may be conceived as made up of an infinite number of acute triangles whoſe baſes are infinitely ſmall and coincide with the periphery of the circle, and whoſe vertices meet in the center; and the laſt rule for regular polygons will ſerve for obtaining the area of the circle; viz. Multiply the radius of any circle (which is half the diameter) by half the periphery of the circle, and the product is the area.

Now by ſeveral operations and tedious proceſſes, the peripherys are found in proportion to the diameters of circles very near, as 3.1416 to 1. Hence it appears that the area of that circle whoſe diameter is multiply'd by half its circumference is equal to .7854; and the area of every circle bears proportion to the ſquare of its diameter as .7854 does to 1; ſo that if you multiply the ſquare of any circle's diameter by .7854, the product will give the area required.

Note. The ſame rules ſerve for a ſemicircle, or quadrant, by taking the parts accordingly.

A Circle.



$\frac{1}{2}$)11.12

5.56
1.77

3892
3892
556

9.8412 Area.

$\frac{1}{2}$)3.54

1.77

Again.

3.54
3.54
1416
1770
1062
12.5316
.7854
501264
626580
1002528
877212
9.84231864 Area.

Again, by croſs-multiplication.

.12
12
1.44 inches.
12
5.28 parts.

.54
12
6.48
12
5.76

F. I. P.
½)3 — 6 — 5
1 — 9 — 2

F. I. P.
½)11 — 1 — 5
5 — 6 — 8

F. I. P.
5 — 6 — 8
1 — 9 — 2
5 — 6 — 8
45 — 54 — 72
10 — 12 — 16
Area 9 — 9 — 7 — 1 — 4

Here

Here follows ſeveral *uſeful propoſitions* about a Circle.

The diameter of a circle being given, to find the circumference.

As 1 is to 3.1415927, ſo is the diameter to the circumference.

The circumference being given to find the diameter.

As 3.1416 is to 1, ſo is the circumference to the diameter.

The circumference being given to find the area.

As 1 is to .0795775, ſo is the ſquare of the circumference to the area required.

By the area of a circle to find the diameter.

As 1 is to 1.2732395, ſo is the content to the ſquare of the diameter.

By the area of a circle to find the ſide of a ſquare equal to it.

Extract the ſquare root of the area.

By the area of a circle to find the circumference.

As 1 is to 12.5663706, ſo is the area to the ſquare of the circumference.

Having the circumference of a circle to find the ſide of the inſcribed ſquare.

As 1 is to .2250791, ſo is the circumference to the ſide inſcribed.

Having the area of a circle to find the area of the inſcribed ſquare.

As 1 is to .6366197, ſo is the area of the circle drawn about the ſquare, to the inſcribed ſquare.

PROBLEM IV. *Of an Ellipſis.*

An ellipſis or oval bears the ſame proportion to its circumſcribing parallelogram as the circle does to its circumſcribing ſquare. Therefore multiply the tranſverſe by the conjugate diameter, and that product again by .7854 for the area.

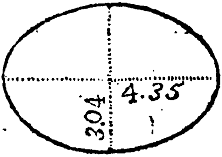
Tranſverſe diameter 4.35
Conjugate diameter 3.04

1740
13050

13.2240
.7854

528960
661200
1057920
925680

Area 10.38612960



PROBLEM V. *Of a Parabola.*

A parabola is ⅔ of the parallelogram circumſcribing it. Therefore multiply the greateſt ordinate by ⅔ of the correſponding abſciſſa, and the product is the area required.

1.71
2

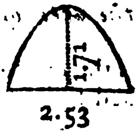
3)3.42

1.14

2.53
1.14

1012
253
253

Area 2.8842

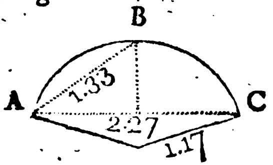


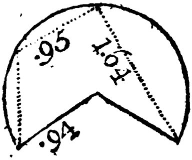
PRO-

PROBLEM VI. *Of a Sector.*

A ſector is equal to a triangle whoſe baſe is equal to the ſectoral arch, and perpendicular altitude equal to the radius which deſcribes that arch. Therefore multiply the radius by half the arch-line for the area of the ſector.

But firſt to find the length of the arch-line, multiply the chord of half the ſegment A B by 8, and from the product ſubtract the chord of the whole ſegment A C, divide the remainder by 3, and the quotient is the arch-line A B C ſought.





1.33
8

10.64
2.27

3)8.37

2.79 Arch Line.

½)2.79

1.395
1.17

9755
1395
1395

1ſt. Area 1.63205

.95
8

7.60
1.64

3)5.96

1.986
.94

7946
17880

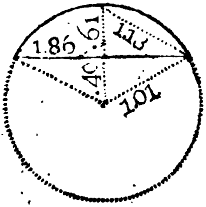
2d. Area 1.86746

N. B. There being two ſuch-like arch-lines in the 2d. ſector as in the 1ſt. in that caſe you take the whole inſtead of half.

PROBLEM VII.

To find the area of the ſegment of a Circle.

Take the difference between the area of the ſector under the ſame arch with the ſegment, and the area of the triangle under the chord and radius, for the area of the ſegment.



½).40
.2

1.86
.2
.372

1.13
8
9.04
1.86
3)7.18
½)2.393
1.196
1.01
1196
119666
1.20863
.372
Area .83663

Note. It may ſometimes happen that you may meet with mixt or compound figures, ſuch as are compoſed of rectilineal and curvileneal figures together: To find the area of ſuch, you muſt firſt find the area of the ſeveral figures of which the whole compound figure is compoſed, and add all the areas together, the ſum will be the area of the whole compound figure.

Some general rules for finding the ſuperficial content of round or ſquare figures, &c.

For round pillars, multiply the length by the circumference.

For ſquare pillars, add the ſides or breadths together, and multiply the total by the length.

For cones, multiply half the length by the largeſt circumference.

For pyramids, add all the breadths at the baſe together, and multiply half the length by the total.

For globes, multiply the area of the greateſt circle by 4, it gives the content.

STEREOMETRY,

OR

MENSURATION OF SOLIDS,

TEacheth how to meaſure and give up the content of all ſolid bodies, which are ſuch as do conſiſt of length, breadth, thickneſs, &c.

PROBLEM I.

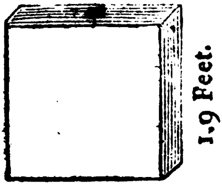
To find the ſolidity of any Cube, Priſm, or Cylinder.

A priſm, &c. is a ſolid body conſtituted of an infinite ſeries of equal areas, that of its baſe being one of the terms, and its height the number of all the terms; hence therefore may be deducted the following rule.

Multiply the area of the baſe by the depth or perpendicular altitude, and the product ſhall be the ſolid content.

Note. In the following examples, the ſolid figures are formed more to anſwer the purpoſe they are deſign'd for, than for perſpectives; as alſo there is this advantage, that the learner may very eaſily form an idea to make others by them.

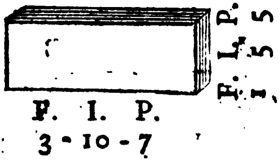
Example of a Cube. Feet.



1.9
1.9
171
19
3.61
1.9
3249
361

Solid content 6.859

Admit the following a ſquare Priſm, viz. more in length than the cube; what's the content in feet, inches, &c.?



F. I. P.
1 — 5 — 5
1 — 5 — 5

1 — 5 — 5
5 — 25 — 25
5 — 25 — 25

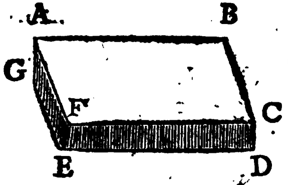
2 — 1 — 3 — 4 — 1
Length 3 — 10 — 7

6 — 3 — 9 — 12 — 3
20 — 10 — 30 — 40 — 10
14 — 7 — 21 — 28 — 7

Anſw. 8 — 2 — 11 — 6 — 7 — 2 — 7

Let A B C D E F G repreſent a Parallelopipedon, whoſe length A B is 3.88 feet, breadth B C 1.86, and depth 1.04 feet; what's the ſolid content?

Joh

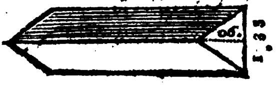


(12)

Feet.
1.86
1.04
―――――
744
1860
―――――
1.9344
3.88
―――――
154752
154752
58032
―――――
Facit 7.505472

A

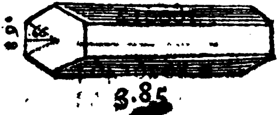
A triangular Priſm.



$\frac{1}{2}$).90
.45

1.35
.45
675
540
.6075
4.12
12150
6075
24300
2.502900 content.

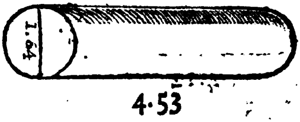
A Priſm of ſix ſides.



.68
.66
408
408
.4488
3 half the ſides.
1.3464
3.85
67320
107712
40392
5.183640 content.

A

A Cylinder.



```
       1.54
       1.54
    -------
        616
       770
      154
  ---------
     2.3716
      .7854
  ---------
      94864
    118580
   189728
  166012
  ---------
 1.86265464
       4.53
  ---------
  558796392
 931327320
745061856
-----------
8.4378255192 ſolid content.
---------------------------
```

The

The ſame by croſs-multiplication.

	F.	I.
½)	4	10
	2	5

The dimenſions in feet, inches, &c. thus; 4 feet 10 inches in circumference, 1 foot 6 inches and 6 parts diameter, and depth 4 feet 6 inches and 5 parts.

	F.	I.	P.			
½)	1	6	6			
	0	9	3			
	2	5				
	0	18	6			
		0	45	15		
	1	10	4	3		
	4	6	5			
	4	40	16	12		
		6	60	24	18	
			5	50	20	15
Content	8	5	4	5	3	3

The cuſtomary method of meaſuring circular ſolids is to girt them about the middle with a ſtring, and take ¼ of that girt for the ſide of a ſquare, by which they meaſure the piece as if it was ſquare; but it is eaſy to be demonſtrated that the fourth part of the circumference of a circle cannot be equal to the ſide of a ſquare of equal area to that circle, it being leſs. For inſtance,

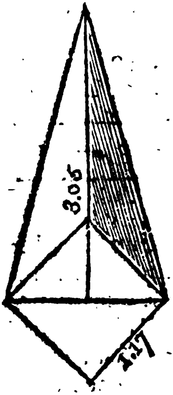
ſtance, in the laſt example, the 4th part of the circumference is 1 foot, 2 inches and an half, which being ſquared and multiplied by the length will give a ſmall matter under 6 feet 8 inches for the content; and thus you ſee the difference of the two methods.

PROBLEM II.

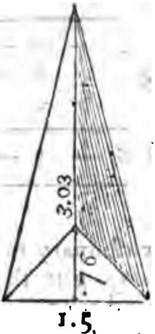
To find the content of any Pyramid or Cone.

A pyramid is ⅓ of its circumſcribing priſm, and a cone is ⅓ of its circumſcribing cylinder. Therefore, multiply the area of the baſe by ⅓ of the perpendicular altitude, and the product will be the content: Or 'twill be the ſame thing if you multiply the area of the baſe by the whole height, and take ⅓ of the product for the content.

EXAMPLES.



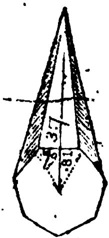
1.17
1.17
819
117
117
1.3689
3.05
68445
410670
⅓)4.175145
Content 1.391715

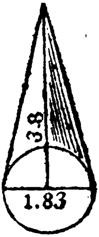


1.5
.38
120
45
.570
1.01
570
5700
Content .57570

½).76
.38

3)3.03
1.01





A Cone.

½).81
.405
.79
3645
2835
.31995
7
2.23965
3.7
1567755
671895
⅓)8.286705
2.762235 content.

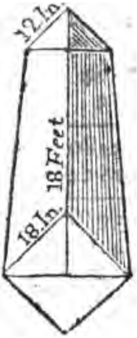
1.83
1.83
549
1464
183
3.3489
.7854
133956
167445
267912
234423
2.63022606
.38
2104180848
789067818
⅓)9.994859028
Content of the cone 3.331619676

PROBLEM III.

To find the content of the fruſtum of a Pyramid or Cone.

DEFINITION. A fruſtum of a pyramid, &c. is the remaining part when the top is cut off parallel to the baſe.

RULE. Find the area of both the greater and leſſer ends; then find a geometrical mean betwixt thoſe areas, which three numbers add together, and multiply by $\frac{1}{3}$ of the depth for the content.



$\frac{1}{3}$)18
6

12
12
144

18
18
144
18
324
144
1296
1296
324
46656(216
4
41)66
41
426)2556
2556

216
324
144
144 {12)684
12)57
4.75
6

Content in feet 28.50

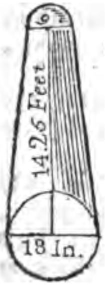
Note. The reaſon of dividing by 144 is, that the baſes are taken in inches, and the length in feet; which if all had been taken in inches, and you would have the anſwer in feet, the laſt operation would be to divide the reſult by the ſolid inches in a foot, inſtead of the ſquare inches.

The uſual way to meaſure ſuch ſolids, is to take a ſquare about the middle of the piece, which they take to be a mean ſquare: This way when the piece is pretty near as thick at one end as at the other, is ſomething near the truth; but when there is a great diſproportion between the ends of the piece, the error is conſiderable, and falls ſhort of the true content.

Of the fruſtum of a Cone.

The rule is the ſame in effect as the fruſtum of a pyramid, but the following may prove ſomewhat ſhorter for the fruſtum of a cone than the former; viz. To the rectangle or product of the diameter of the two baſes, add the ſquares of the ſaid diameters, and multiply the ſum by .7854, the product will be the triple of a mean area, which multiply'd by $\frac{1}{3}$ of the perpendicular height will give the ſolid content.

See the work in the next page.



```
 9            18          18
 9             9          18
---          ---         ---
81           162 ⎫       144
             324 ⎬       18
              81 ⎭       ---
             ---         324
             567
            .7854
            ----
            2268
           2835
          4536
         3969
        ---------               1/3)14.25
     ⎧ 12)445.3218                  -----
  144⎨    ---------                 4.75
     ⎩ 12)37.11015
          ---------
          3.0925125
               4.75
         ----------
          154625625
         216475875
        123700500
         ----------
Solid content 14.689434375
```

The common method uſed in meaſuring theſe ſolids, is to girt them about the middle between both ends, and take a 4th of the compaſs and eſtimate it as though it was the ſide of a ſquare, as in the remark on the cylinder. The method is beſt adapted to the meaſuring of timber trees, and round timber for ſale, and ſeems chiefly deſign'd for that purpoſe, as ſomething conſiderable ought to be allowed for waſte, &c. (beſide the bark) before ſuch timber can be fit for uſe; but in other matters where it is required to be more exact, the loſs is great.

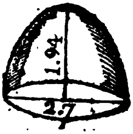
The greateſt error is, that all circular pieces are in general meaſured by this method, either on account of its eaſe, or through ignorance in knowing no truer way: But let the following remark explain the nature of this cuſtomary method uſed in rough timber for ſale; that is, meaſurers and workmen reckon 40 feet of unhewn or rough timber to the load, ſuppoſed to weigh about a ton, or 20 hundred weight: For as hewn timber is meaſured by the ſquare, it is near exact; but rough timber is meaſured by the girt, or quarter compaſs, and is about $\frac{1}{5}$ leſs than exact. Therefore in buying of timber, it amounts to much the ſame, whether it be meaſured by the quarter compaſs at 40 feet ſolid to a load, or meaſured exact at 50 feet to a load.

PROBLEM IV.

To find the content of a Parabolic Conoid.

RULE. Multiply the area of the baſe by half the height for the ſolidity.

See the work in the next page.



2)1.94
.97

2.7
2.7
189
54
7.29
.7854
2916
3645
5832
5103
5.725566
.97
40078962
51530094
Content 5.55379902

Note. A parabolic conoid is $\frac{1}{2}$ of its circumſcribing cylinder; and an hyperbolic conoid (being much in the ſame form) is $\frac{5}{12}$ of its circumſcribing cylinder: The difference lies in the curve towards its baſe, that of the hyperbolic kind being leſs curved than the other as it approaches its baſe, and therefore falls off from its center, having the outſide lines more ſtraight.

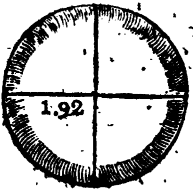
PROBLEM V.

To find the content of a Sphere, &c.

A ſphere or globe is a round ſolid body, and is $\frac{2}{3}$ parts of its circumſcribing cylinder. Therefore, multiply the area of that circle whoſe diameter is equal to the globe's axis with $\frac{2}{3}$ of the globe's axis for the content. Or if you multiply the cube of the diameter by .5236, that will give the ſolid content. Otherwiſe, multiply the axis or diameter into the circumference, the product is the ſuperficial content; which multiplied by a ſixth part of the axis, gives the ſolidity.

A ſpheroid bears the ſame proportion to its circumſcribing cylinder as the ſphere does; therefore multiply the area of the conjugate axis by $\frac{2}{3}$ of the tranſverſe, and the product is the content of the ſpheroid.

A Sphere.



1.92
1.92

384
1728
192

3.6864
1.92

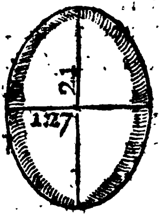
73728
331776
36864

7.067888
.5236

42407328
21213664
14135776
35339440

3.7007561568
Content of the ſphere.

A Spheroid.



2.1
2
---
3)4.2

1.4

1.27
1.27

889
254
127

1.6129
.7854

64516
80645
129032
112903

1.26677166
1.4

506708664
126677166

1.773480324
Content of the ſpheroid.

Here

Here follows ſeveral *uſeful propoſitions* about the Sphere.

Having the axis of a globe, or diameter, to find the ſuperficial content.

As 1 is to 3.1415927, ſo is the ſquare of the axis to the ſuperficial content.

By the circumference of a globe, to find the ſuperficial content.

As 1 is to .31832, ſo is the ſquare of the circumference to the ſuperficial content.

By the circumference of a globe, to find the ſolid content.

As 1 is to .016887, ſo is the cube of the circumference to the ſolid content.

Having the ſolid content of a globe, to make a cube equal to the globe.

Extract the cube root out of the ſolid content of the globe.

By the axis of a globe, to make a cube equal to the ſolid content thereof.

As 1 is to .86604, ſo is the axis to the cube root, &c.

By the circumference of a globe, to make a cube equal to the ſolid content.

As 1 is to .256556, ſo is the circumference to the cube root, &c.

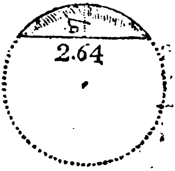
By the axis of a globe, to make a ſquare equal to the ſuperficial content of the globe.

As 1 is to 1.77254, ſo is the axis to the ſquare root, &c.

PROBLEM VI.

To find the content of the ſegment of a Sphere.

RULE. To three times the ſquare of half the ſegment's baſe, add the ſquare of the height or verſed ſine; multiply this ſum by the verſed ſine, and that product by .5236; and this laſt product is the content.



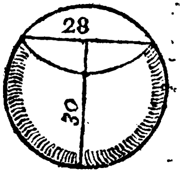
```
                    ½)2.64
                      ----
                      1.32
                      1.32
                      ----
                       264
                      396
                     132
                    ------
                    1.7424
                         3
                    ------
                    5.2272
                     .4489
                    ------
  .67               5,6761
  .67                  .67
  ---              -------
  469               397327
 402               340566
 -----            --------
 .4489            3.802987
                     .5236
                  --------
                  22817922
                 11408961
                7605974
               19014935
               ------------
       Content 1.99124339932
```

PRO-

PROBLEM VII.

To find the content of the fruſtum of a Sphere.

RULE. To the area of the fruſtum's diameter, add the area of the depth, more one third of the ſame area; multiply the ſum by half the depth for the content.



28
28

224
56

784
.7854

3136
3920
6272
5488

615.7536
1/3)706.86
235.62

1558.2336
15

77911680
15582336

Content 23373.5040

30
30

900
.7854

706.8600

1/2)30

15

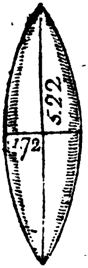
PRO-

PROBLEM VIII.

To find the content of a Parabolic Spindle.

× Cylinder

A parabolic ſpindle is $\frac{8}{15}$ of its circumſcribing× ~~ſpindle~~: Therefore, multiply the area of the diameter with $\frac{8}{15}$ of the greateſt ordinate for the content.



5.22
8

15)41.76(2.78
30

117
105

126
120

6

1.72
1.72

344
1204
172

2.9584
.7854

118336
147920
236672
207088

2.32352736
2.78

1858821888
1626469152
464705472

Solid content 6.4594060608

N. B. The ſolidity of any irregular body may be determined by immerſing the ſame in a veſſel of water; for the ſolid content of the additional ſpace occupied by the fluid on account of the immerſed body, will be equal to the ſolidity of that body; and thus may the ſolidity of ſtatues, &c. be very exactly determin'd.

Here follows ſome various Examples *in* MENSURATION.

There is a court whoſe length is $21\frac{1}{2}$ feet, and breadth $17\frac{1}{2}$, which is to be pav'd with ſtones each 18 inches ſquare; how many ſuch ſtones will pave it?

```
            21.5
     1.5    17.5
     1.5    ————
    ————    107
     75    1505
     15     215
  ——————   ————
  2.25)376.25(167  Anſwer.
        225
       ——————
        1512
        1350
       ——————
         1625
         1575
       ——————
           50
```

If a board be 19 inches broad, how much in length will make a ſquare foot?

12
12
—— Inches.
19)144(7.57
133 Anſwer.
———
110
95
———
150
133
———
17

How many ſquare rods are there in a wall which is 87¼ feet long, 12½ feet high, and 2½ bricks thick?

Note. Brickwork is generally eſtimated by the rod ſquare, which contains 272¼ ſquare feet; but for practice 272. Alſo bricklayers compute or value their work at the rate of a brick and an half thick; and if the thickneſs of the wall ſhould be more or leſs than ſuch, it is reduced to that thickneſs by multiplying the area of the wall, by the number of half bricks the thickneſs of the wall is of, and dividing the product by 3.

See the work in the next page.

```
           87.25
            12.5
          ------
           43625
          17450
          8725
        --------
   272)1090.625 (   4.009
       1088             5
   ------------     -------
           2625   3)20.045
           2448     -------
          -----      6.6816 Anſwer.
            177     --------
```

To find how much in length makes a ſolid foot of any ſquared or regular timber, (i. e.) *ſuch as have equal baſes.*

Rule. Always divide 1728 (the ſolid inches in a foot) by the area of the baſe; the quotient is the length of the foot.

If a piece of timber be 22 inches deep, and 15 inches broad, how much in length will make a ſolid foot?

```
                         Inches.
                330)1728(5.23
                    1650   Anſwer.
                    -------
     22              780
     15              660
    ----            -----
    110             1200
    22               990
    ----            -----
    330              210
```

Admit a ſquare piece of timber in form of the fruſtum of a pyramid 12 feet long, the ſide of the greater baſe 21 inches, and the ſide of the leſſer baſe 3 inches; how far muſt be meaſur'd from the greater end to cut off 5 ſolid feet?

10.5 half the greater ſide.

1.5 half the leſſer ſide.

—— F.

As 9 : 12 : : 10.5

12

9)126.0

14 { Feet the whole length of the pyramid.

21

21

21

42

441

14

1764

441

⅓)6174

144)2058(14.3 Solid content of the whole pyramid (very near.)

144 5

618 9.3 The remaining part of the whole pyramid when the 5 ſolid feet are cut off from the greater end.

576

420

432

See part of the work in the next page.

14
14

56
14

196
14

784
196

As 14.3 : 2744 : : 9.3
9.3

8232
24696

14.3)25519.2(1784.559
143

1121
1001

1209
1144

652
572

800
715

850
715

1350
1287

63

See the remainder of the work in the next page.

1784.559(14.
12.129

1.871

1

364)784
728

43561)56559
43561

4399564)12998000
8799128

441010521)4198872000
3969094689

229777311

F.
Anſwer 1.871
12

viz. 22.452. inches.

N. B. I have not any where treated of giving up the content by Scale and Compaſſes, nor have I as yet made any mention of the Sliding-Rule, the treating of which I have reſerved for the following article of Gaging; having each example work'd both by pen and ſliding-rule; ſo that when a perſon comes to underſtand the nature of the ſlide, he may make uſe of it in any other practical caſe as well as in gaging; for the improvements on the ſlide are now become very uſeful, and are far more expeditious and convenient than that antique method of computing the content by ſcale and compaſſes.

GAGING.

GAGING.

THAT gaging is included in the ſcience of Stereometry is very plain to be underſtood, as the capacities or contents of all ſorts of veſſels for liquors, &c. are computed as tho' they were really ſolid bodies.

And herein I preſuppoſe that the learner is already well acquainted with the principal rules of arithmetic, eſpecially in multiplication and diviſion, both in whole numbers and decimal parts; and he ſhould likewiſe underſtand the rule of proportion. Alſo he ought to have ſo much skill in ſolids as to determine at ſight what ſort of figure any utenſil is of, or what figure, &c. it may be reduced to, ſo that its dimenſions may be taken, and the content thereof computed as near as poſſible; for there is not any tun or cask, &c. ſo regularly made, as by the rules of art 'tis requir'd to be. Likewiſe the learner muſt know, that all dimenſions in gaging are taken in inches and decimal parts of an inch; for the content of a gallon or buſhel, &c. is known by the ſtandard of its kind to be a certain number of cubic inches, &c. which I ſhall further acquaint my pupil with. And he muſt alſo know that in the buſineſs of gaging, all ſuperficies or areas are always underſtood to be one inch deep, otherwiſe it could not be ſaid (as in the gager's language it is,) that the area of ſuch a ſquare, or ſuch a circle, &c. is ſo many gallons, &c.

Note. The barrel in the exciſe is reckon'd at 34 gallons, both for ale and beer.

I ſhall firſt begin with finding the areas of ſuperficies in ale and wine gallons, and in malt buſhels.

Example 1ſt. Suppoſe a ſquare (which has all its ſides equal,) and the ſide 45.5 inches; what will its area be in ale and wine gallons, and in malt buſhels?

Rule. Multiply the length or breadth (being here equal) into itſelf, and the product will be the area in inches; then divide by the cubic inches in the reſpective gallon, &c. and the quotient will be the area requir'd.

$$\text{Cubic inches.}\left\{\begin{array}{l}\text{282 in a ale gallon.}\\ \text{231 in a wine gallon.}\\ \text{2150 in a malt buſhel.}\end{array}\right.$$

$$\begin{array}{r}
45.5\\
45.5\\
\hline
2775\\
2275\\
1820\\
\hline
282)2070.25(7.34\\
1974\\
\hline
962\\
846\\
\hline
1165\\
1128\\
\hline
37
\end{array}$$

See the remainder of the work in the next page.

```
231)2070,25(8.96 Wine.
    1848
    ----
     2222                    2150)2070.25(.96
     2079                         19350      Malt.
     ----                         -----
      1435                         13525
      1386                         12900
      ----                         -----
        49                           625
```

The ſame by the Sliding-Rule.

Set each proper diviſor upon A to a ſide of the ſquare on B, then againſt the other ſide of the ſquare on A is the area on B:

		A		B		A		B	
Viz.	As	282	:	45.5	::	45.5	:	7.34	ale gall.
	As	231	:	45.3	::	45.5	:	8.96	wine ga.
	As	2150	:	45.5	::	45.5	:	.96	of a malt buſhel.

EXAMPLE 2d. Let the length of a tun, back, cooler, ciſtern, couch, &c. be 145.3 inches, and mean breadth 88.6 inches; what is the area in ale, wine, and malt?

RULE. Multiply the length by the breadth, and divide by the cubic inches, &c.

See the work in the next page.

145.3
88.6

8718
11624
11624

282) 12873.58(45.65
1128 Ale.

1593
1410

1835
1692

1438
1410

28

231)12873.58(55.72
1155 Wine.

1323
1155

1685
1617

688
462

226

2150)12873.58(5.98
10750 Malt.

21235
19350

18858
17200

1658

Same by the Slide.

Set each proper diviſor on A to the length on B, then againſt the breadth on A is the area on B:

		A		B		A		B	
Viz.	As	282	:	145.3	::	88.6	:	45.65	ale.
	As	231	:	145.3	::	88.6	:	55.72	wine.
	As	2150	:	145.3	::	88.6	:	5.98	malt.

Note. When one length or breadth will not ſerve, find a mean length or breadth thus; viz. take them in ſeveral places, and divide the total by the number of times taken.

Example 3d. If the length of the baſe of a triangle be 60 inches, and its perpendicular height 36 inches, what will its area be in ale, wine, and malt?

Rule. Multiply the baſe into half its perpendicular, and divide by the cubic inches, &c.

½)36

18
60

282)1080(3.8
846 Ale.

2340
2256

84

231)1080(4.6
924 Wine.

1560
1386

174

2150)1080.0(.5
10750 Malt.

50

By

By the Slide.

Set each proper diviſor on A to the baſe on B, then againſt half the perpendicular on A is the area on B:

A B A B
Viz. { As 282 : 60 :: 18 : 3.8 ale.
As 231 : 60 :: 18 : 4.6 wine.
As 2150 : 60 :: 18 : .5 malt.

Note. By the ſame rule, you may eaſily find the area of any trapezium, or polygon, &c. Firſt divide it into triangles, and then find the areas of thoſe triangles in inches; the ſum of thoſe areas being divided by its proper diviſor, will give the area requir'd. But if it be a trapezium, one operation may ſerve.

EXAMPLE 4th. Suppoſe the diameter of a circular veſſel to be 45.5 inches; what is the area in ale, wine, and malt?

RULE. Divide the ſquare of the diameter by theſe circular diviſors, viz. 359 for ale, and 294 for wine gallons; and 2737.47 for malt buſhels.

See the work in the next page.

```
       45.5          294)2070.25(7.04
       45.5              2058      Wine.
     ------             -----
       2275              1225
      2275               1176
     1820               -----
     ------                49
359)2070.25(5.76
    1795      Ale.
    -----            2737.47)2070.2500(.75
     2752                    1916229   Malt.
     2513                   --------
    -----                    1540210
     2395                    1368735
     2154                   --------
    -----                     171475
      241
```

By the Slide.

Set each of the above diviſors on A to the diameter on B, then againſt the ſaid diameter on A is the area on B:

		A		B		A		B	
Viz.	As	359	:	45.5	::	45.5	:	5.76	ale.
	As	294	:	45.5	::	45.5	:	7.04	wine.
	As	2737.47	:	45.5	::	45.5	:	.75	malt.

EXAMPLE 5th. Let the tranſverſe diameter of an ellipſis be 55 inches, and conjugate diameter 45 inches; what's the area in ale, wine, and malt?

RULE. Multiply the tranſverſe diameter by the conjugate, and divide by the circular diviſors.

See the work in the next page.

55
.45
275
220
359)2475(6.8
2154 Ale.
3210
2872
338

294)2475(8.4
2352 Wine.
1230
1176
54

2737.47)2475.000(.904
2463723 Malt.
1127700
1094988
32712

By the Slide.

Set each proper diviſor on A to the tranſverſe diameter on B; then againſt the conjugate diameter on A is the area on B:

		A		B		A		B	
Viz.	As	359	:	55	: :	45	:	6.8	ale.
	As	294	:	55	: :	45	:	8.4	wine.
	As	2737.47	:	55	: :	45	:	.904	malt.

The following Examples are to compute the contents of all ſolid bodies uſed in gaging.

Example 6th. Suppoſe a cube whoſe every ſide is 24 inches; what's the content in ale, wine, and malt?

Rule. Cube the ſide and divide by the ſquare diviſors.

See the work in the next page.

24
24
96
48
576
24
2304
1152
282)13824(49 Ale.
1128
2544
2538
6

231)13824(59.8 Wine.
1155
2274
2079
1950
1848
102

2150)13824(6.42 Malt.
12900
9240
8600
6400
4300
2100

The ſame by the Slide.

Upon the *Rule* you'll find theſe ſquare gage-points on the ſide D, being the ſquare roots of the ſolid inches in each reſpective gallon or buſhel, viz. 16.79 for ale; 15.19 for wine; and 46.36 for malt. Then ſet each gage-point on D to 24 on C, and againſt 24 on D is the content on C:

		D		C		D		C	
Thus	As	16.79	:	24	: :	24	:	49	ale gall.
	As	15.19	:	24	: :	24	:	59.8	wine gall.
	As	46.36	:	24	: :	24	:	6.42	malt buſh.

Example 7th. Admit a priſm in form of a parallelopipedon as follows; 45.6 inches in length, breadth 27.5 inches, and depth 21.5 inches; what's the content in ale, wine, and malt?

Rule. Multiply the length by the breadth, and that product by the depth, and divide by the ſquare diviſors.

```
      45.6              231)26961.0(116.7
      27.5                  231          Wine.
      ----                  ---
      2280                  386
     3192                   231
     912                    ---
     -------                1551
     1254.00                1386
       21.5                 ----
     -------                 1650
      6270                   1617
     1254                    ----
    2508                       33

 282)26961.0(95.6       2150)26961.0(12.54
     2538      Ale.          2150       Malt.
     ----                    ----
     1581                    5461
     1410                    4300
     ----                    -----
     1710                    11610
     1692                    10750
     ----                    -----
       18                     8600
                              8600
                              ----
```

To work the ſame by the Slide.

Firſt find a mean proportional betwixt the length and the breadth thus; ſet either of the two numbers given upon D to the ſame on C, then againſt the other number on D is the mean requir'd on C;

$$\begin{array}{lccccl} & \text{D} & & \text{C} & & \text{D} & & \text{C} \\ \text{viz. As } & 27.5 & : & 27.5 & :: & 45.6 & : & 35.4 \text{ mean length.}\end{array}$$

Then ſet each proper gage-point for a ſquare to the depth on the lines C and D, and againſt your mean is the content.

$$\begin{array}{lcccccccl} & \text{D} & & \text{C} & & \text{D} & & \text{C} & \\ \text{Thus}\left\{\begin{array}{l}\text{As}\\ \text{As}\\ \text{As}\end{array}\right. & \begin{array}{c}16.79\\ 15.19\\ 46.36\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \begin{array}{c}21.5\\ 21.5\\ 21.5\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \begin{array}{c}35.4\\ 35.4\\ 35.4\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \begin{array}{c}95.6\\ 116.7\\ 12.54\end{array} & \begin{array}{l}\text{ale.}\\ \text{wine.}\\ \text{malt.}\end{array}\end{array}$$

Or otherwiſe you may find the content upon the ſide of the *Rule* A and B; viz. after finding the area, ſet 1 on A to the area on B, then againſt the depth on A is the content on B.

$$\begin{array}{lcccccccl} & \text{A} & & \text{B} & & \text{A} & & \text{B} & \\ \text{Thus}\left\{\begin{array}{l}\text{As}\\ \text{As}\\ \text{As}\end{array}\right. & \begin{array}{c}282\\ 231\\ 2150\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \begin{array}{c}45.6\\ 45.6\\ 45.6\end{array} & \begin{array}{c}::\\ ::\\ ::\end{array} & \begin{array}{c}27.5\\ 27.5\\ 27.5\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \left.\begin{array}{c}4.44\\ 5.42\\ .58\end{array}\right\} & \text{areas.}\end{array}$$

$$\begin{array}{lccccccc} \text{Then}\left\{\begin{array}{l}\text{As}\\ \text{As}\\ \text{As}\end{array}\right. & \begin{array}{c}1\\ 1\\ 1\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \begin{array}{c}4.45\\ 5.42\\ .58\end{array} & \begin{array}{c}::\\ ::\\ ::\end{array} & \begin{array}{c}21.5\\ 21.5\\ 21.5\end{array} & \begin{array}{c}:\\ :\\ :\end{array} & \left.\begin{array}{c}95.6\\ 116.7\\ 12.54\end{array}\right\}\text{contents.}\end{array}$$

Note. From the foregoing examples it will be obvious how to find the content of any ſquare ſolid or regular priſm with more or leſs ſides; viz. to multiply the area of the baſe by the depth and divide by the ſquare diviſors, &c.

Now upon the bottom of the *Rule*, under the lines A and B, is a reverſe line, mark'd M D, ſignifying malt depth; ſo that if you ſet the length upon M D to the breadth on B, then againſt the depth on A is the content on B.

Example. Let the length of a couch be 255 inches, breadth 161, and depth 7.5; what's the content in buſhels?

	MD		B		A		B	
As	255	:	161	: :	7.5	:	143.2	Anſwer.

Example 8th. Suppoſe a veſſel in form of a cylinder, whoſe diameter is 38.7 inches, and the depth 59.5 inches; what's the content in ale, wine, and malt?

Rule. Multiply the ſquare of the diameter by the depth, and divide by the circular diviſors.

See the work in the next page.

```
      38.7
      38.7
    ------
      2709
     3096
    1161
  ---------
   1497.69
      59.5
  ---------
    748845
   1347921
   748845
  ---------
359)89112.555(248.2
    718          Ale.
    -----
    1731
    1436
    -----
     2952
     2872
     -----
      805
      718
     -----
       87
```

```
294)89112.5(303.1
    882         Wine.
    ----
      912
      882
     -----
      305
      294
     -----
       11

2737.47)89112.555(32.5
        821241     Malt.
        -------
         698845
         547494
        --------
        1513515
        1368735
        --------
         144780
```

The ſame by the Slide.

Upon the *Rule*, on the ſide D, is the following circular gage-points, (which are the ſquare roots of their proper diviſors) viz. 18.94 for ale, 17.14 for wine, and 52.32 for malt. Then ſet each proper gage-point on D to the depth on C, and againſt the diameter on D is the content on C: Thus

D C D C

Thus { As 18.94 : 59.5 :: 38.7 : 248.2 ale.
As 17.14 : 59.5 :: 38.7 : 303.1 wine.
As 52.32 : 59.5 :: 38.7 : 32.5 malt.

EXAMPLE 9th. Admit a tun or veſſel, &c. in form of the fruſtum of a cone, thus; the greater diameter 40 inches, the leſſer diameter 30, and the depth 60 inches; what's the content in ale gallons?

RULE. Find the area of each baſe, and a mean proportional between them, multiply the ſum of thoſe three by one third part of the depth or height, and the product is the content.

30	40
30	40
359)900(2.506	359)1600(4.456
718 Leſſer area.	1436 Greater area.
1820	1640
1795	1436
2500	2040
2154	1795
346	2450
	2154
	296

See the remainder of the work in the next page.

4.456
2.506
26736
222800
8912
11.166736(3.341
9
63)216
189
664)2767
2656
6681)11136
6681
4455

⅓)60
20

2.506
4.456
3.341
10.303
20
Anſ. 206.060 ale gall.

The ſame by the Slide.

	D		C		D		C	
As	18.94	:	1	: :	30	:	2.506	leſſer area.
As	18.94	:	1	: :	40	:	4.456	greater ar.
As	2.506	:	2.506	: :	4.456	:	3.341	mean proportional.

10.303
20
Ale gallons 206.060

EXMA-

Example 10th. Suppoſe the dimenſions of a tun or ciſtern in form of the fruſtum of a pyramid, as follows; the length of the greater baſe 96 inches, and its breadth 65.3; the length of the leſſer baſe 75.8, and its breadth 42.7 inches; and the depth 40.35 inches; what's the content in malt buſhels?

65.3
96
3918
5877
2150)6268.8(2.915 Greater area.
4300
19688
19350
3380
2150
12300
10750
1550

75.8
42.7
5306
1516
3032
2150)3236.66(1.505 Leſſer area.
2150
10866
10750
11660
10750
910

See the remainder of the work in the next page.

2.915
1.505

14575
145750
2915

4.387075(2.094
4 Mean.

409)3870
3681

4184)18975
16736

2239

6.514
13.45

32570
26056
19542
6514

Anſwer 87.6130 buſhels.

⅓)40.35

13.45

2.915
1.505
2.694

6.514

The ſame by the Slide.

D C D C
As 46.36 : 65.3 : : 96 : 2.915 greater area.
As 46.36 : 75.8 : : 42.7 : 1.505 leſſer area.
As 2.915 : 2.915 : : 1.505 : 2.094 mean.
This ſum multiply'd by ⅓ of the depth gives the content as before. 6.514

Note. If you wou'd gage a globe, or oval ſolid, or the ſegment or fruſtum of a globe, &c. you may work as in menſuration; they being of little uſe in practical gaging.

A practical rule for finding the content of any conical tun, copper, by-tub, or caſk.

Take the diameter ſomewhat above the middle, if it be broader at top than at bottom, or on the contrary, ſomewhat next to the broadeſt end; and thus it is reduc'd to a cylinder: Then multiply the ſquare of the diameter by the depth, and divide by the circular diviſors, &c.

The following is the practical method of inching, gaging, and fixing any tun, copper, &c. in order for the ready caſting up of each gage.

Whether the ſides are ſtraight or not from top to bottom, you are to take your ſeveral areas as follows; that is, the area to each diameter taken in the middle of every 10 inches from the bottom upwards; thus, the firſt diameter at 5 inches from the bottom, the ſecond at 15 inches, &c. and each of theſe areas ſerve for 10 inches one above the other: A ſpecimen of which is in the next page.

	Depth 43.5	Areas
Diameters	41.4	4.77
	43.8	5.34
	40.6	4.54
	36.0	3.60

See the content upon every inch of the depth, (as fixed in the dimenſion books of exciſe.)

Gall.

Whole content is } 199.195

Depths	Contents	Depths	Contents
1	3.6	23	97.42
2	7.2	24	102.76
3	10.8	25	108.10
4	14.4	26	113.44
5	18.0	27	118.78
6	21.6	28	124.12
7	25.2	29	129.46
8	28.8	30	134.80
9	32.4	31	139.57
10	36.0	32	144.34
11	40.54	33	149.11
12	45.08	34	153.88
13	49.62	35	158.65
14	54.16	36	163.42
15	58.70	37	168.19
16	63.24	38	172.96
17	67.78	39	177.73
18	72.32	40	182.50
19	76.86	41	187.27
20	81.40	42	192.04
21	86.74	43	196.81
22	92.08	43.5	199.195

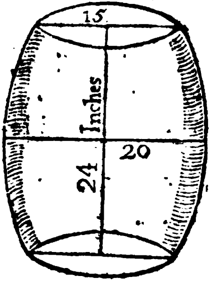
Of

OF CASK GAGING.

To gage a cask in form of the middle fruſtum of a ſpheroid, or thoſe much curved.

RULE. Firſt take the difference betwixt the bung and head diameter, and multiply it by 7 tenths, adding the ſame to the head, and it will reduce the cask to a cylinder: Then multiply the ſquare of the diameter by the length, and divide by the circular diviſors for the content.

Give up the content of the following example of a ſpheroidical cask, in ale and wine gallons.



20
15
5
.7
3.5
15
18.5
18.5
925
1480
185
342.25

342.25
24
136900
68450
359)8214.00(22.88 Ale.
718
1034
718
3160
2872
2880
2872
8

See the remainder of the work in the next page.

294)

294)8214.00(27.93 Wine.
588
2334
2058
2760
2646
1140
882
258

The ſame by the Slide.

D C D C
Thus { As 18.94 : 24 :: 18.5 : 22.8 ale gallons.
As 17.14 : 24 :: 18.5 : 27.9 wine gall.

N. B. There are three ſorts of casks beſide the conical ones, &c.

Viz. {
1ſt. The middle fruſtum of a ſpheroid.
2d. The middle fruſtum of a parabolic ſpindle.
3d. The middle fruſtum of two parabolic conoids abutting upon one common baſe.

There are rules for finding the content of each ſort ſeverally; but in general practice, and particular that of the exciſe, ſuch casks are reſpectively taken in the 1ſt. variety (as is repreſented by the *figure* in the preceding page) ſo that one general rule may ſerve for bulged casks.

If you wou'd gage each ſort ſeverally, the neareſt method is to multiply the difference of the bung and head diameters by .7 for the firſt form, which are thoſe much curved; by .65 for the ſecond form, if the ſtaves be ſomewhat leſs curved; and by .6 for the third form, if the ſtaves between the bung and head be very little curved; then add the product to the head diameter, and the ſum is a mean diameter.

N. B. When you meet with caſks repreſenting the lower fruſtums of two equal cones, you may either multiply the difference of the bung and head diameters by .55 as above, or you may gage them as fruſtums, &c. as in the 9th example.

Note. The diagonal line of the ſpheroidical caſk in the preceding example will be 21.219 inches. Thus another caſk in the ſame form will hold 78 gallons wine meaſure if the dimenſions are as follows; viz. head diameter 24.286 inches, bung diameter 28.334, length 31.225, and diagonal 35.839 inches. Then if you would find the content of any caſk in the ſame form by the diagonal line only, obſerve this proportion; As the cube of 35.839 is to 78 gall. ſo is the cube of any given diagonal to the content of its caſk. For example; Suppoſe a caſk whoſe diagonal is 53.84; ſay, as the cube of 35.839 is to 78 gall. ſo is the cube of 53.84 to 408 wine gallons for anſwer; and ſo with any other caſk of like form.

Again, to gage casks by the Sliding-rule.

Upon the ſide of the *Rule* is plac'd a line of inches, and under it three other lines ſignifying the three varieties, the uſe of which is to reduce a cask to a cylinder.

RULE. Firſt ſubtract the head from the bung diameter, and find the difference on the ſaid line of inches; add that number (which you find againſt ſuch difference) to the head diameter and 'twill reduce the cask to a cylinder: Then ſet the gage-point to the length, and againſt the mean diameter is the content.

EXAMPLE. Given the head diameter 24.5 inches, bung diameter 29.7, and length 45, to find the content in ale and wine gallons?

As 18.94 : 45 : : 28.14 : 99.7 ale gallons.
As 17.14 : 45 : : 28.14 : 121.2 wine gallons

Another Example by the Rule.

Let the head be 19.6, bung diameter 24.5, and length 28.2; what's the content in wine and ale gallons?

As 17.14 : 28.2 : : 23.03 : 50.8 wine gallons.
As 18.94 : 28.2 : : 23.03 : 41.6 ale gallons.

Of ULLAGING *of* CASKS.

Firſt to ullage a ſtanding cask, being part full.

RULE. Divide the wet or dry inches by the cask's length, and if the quotient exceeds .5000, add to the ſaid quotient one tenth part of the exceſs, but if it be under .5000, ſubtract one tenth part of the want; ſo will the ſum or remainder be a decimal number, by which if you multiply the content of the veſſel, the product will be the quantity of liquor therein if your dividend was the wet inches, but the quantity of liquor which is required to fill it up if the dividend was the dry inches.

Obſerve in ullaging, that there muſt be 4 decimal places at leaſt in the quotient.

EXAMPLE. Suppoſe the cask's content to be 61 gallons, whoſe length is 32 inches, 18 inches wet, and 14 dry; what will be the contained liquor, and alſo the vacuity?

```
32)18.0000 ( .5625
    160       .00625 one 10th of the exceſs.
    ———       ——————
    200       .56875
    192           61
    ———       ——————
      80       56875
      64     341250
     ———     ———————
     160    34.69375 contain'd liquor.
     160    ————————————
     ———
```

See the remainder of the work in the next page.

```
                 (.5000
32)14.0000       (.4375
   128            -----
   ---            .0625
    120           -----
     96           .00625 one 10th of the want.
    ---           ------
    240           .43125
    224               61
    ---           ------
     160           43125
     160          258750
     ---          ------
                  26.30625 vacuity.
                  --------

                  34.69375
                  26.30625
                  --------
            Proof 61 gall.
```

To work the ſame by the Sliding-rule.

Upon the ſame ſide of the *Rule* C and D are two lines mark'd with ſegment ſtanding and ſegment lying. Firſt ſet the bung's diameter on C to 100 on ſegment ſtanding, and againſt the wet inches on C is a 4th number, which keep: Secondly ſet 100 upon the line B to the caſk's content on A, then againſt the number laſt found you'll find the quantity of liquor contained.

N. B. By the ſame rule you'll find how much it wants to fill it up if you make uſe of the dry inches.

To

To find the ullage of a cask which lies on one ſide.

RULE. Divide the wet or dry inches by the bung's diameter; if the quotient be above .5000, add to it ¼ part of the exceſs, and multiply that ſum by the cask's content for the liquor in the cask; but if it be under .5000, take from it one fourth part of what it wants, and multiply the remainder by the cask's content for the liquor which is wanted to fill it up.

EXAMPLE. Let the cask be the ſame as before only in another poſition, and not ſo full of liquor; content 61 gallons, bung diameter 28 inches, wet 7 inches, and dry 21: what is the vacuity, and contained liquor?

28)21.0000 (	.7500	
196	.0625	
140	.8125	.7500
140	61	.5000
00	8125	¼).2500
	48750	
		.0625
Vacuity	49.5625	

28)7.0000 (	.2500	
56	.0625	
140	.1875	.5000
140	61	.2500
00	1875	¼).2500
	11250	
		.0625
Liquor in the cask	11.4375	

To perform the ſame by the Slide, follow the directions of the former rule, only make uſe of the length inſtead of the bung, &c.

Note. In ullaging of caſks, the content of the caſk is always made uſe of, whether it be in ale or wine gallons.

To find the content of any hogſhead or caſk of dry goods in feet, inches, &c.

Rule. Take a fourth part of the girt of the bulge, and multiply it by itſelf, which multiply by the length of the veſſel, and to the product add the one fifth thereof, and the ſum is the content.

Example. Let the caſk be 7 feet long, and 9 feet girt, how many feet?

$$\begin{array}{rrrl}
\text{F.} & \text{I.} & & \\
\frac{1}{4})9 - & 0 & & \\
\hline
2 - & 3 & & \\
2 - & 3 & & \\
\hline
4 - & 6 & & \\
 & 6 - & 9 & \\
\hline
5 - & 0 - & 9 & \\
 & & 7 & \\
\hline
\frac{1}{5})35 - & 5 - & 3 & \\
7 - & 1 - & 0, & \&c. \\
\hline
\text{Facit } 42 - & 6 - & 3, & \&c.
\end{array}$$

SURVEY-

SURVEYING OF LAND

IS the application of the rules of Planimetry to the meaſuring and planing the areas of real fields, grounds, &c. by the help of proper inſtruments.

In the following inſtructions for ſurveying of land, we ſhall make uſe of no other inſtrument but the Chain, for that uſed properly is the moſt compleat inſtrument for the purpoſe; far exceeding the beſt plain-tables, theodolites, circumferentors, &c. both in accuracy and expedition, whether the ſurveys are large or ſmall, from a ſingle field to a lordſhip, county or kingdom. Therefore the ſurveyor being furniſhed with a chain only, he may proceed after the following methods, which a little practice will make familiar to him.

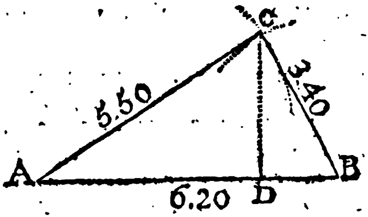
N. B. The chain is 4 poles or 22 yards in length; it is divided into 100 decimal parts or links, each link contains 7.92 inches. And in an acre is contained 10 ſquare chains, (viz. 10 in length and 1 in breadth;) or there are 100000 ſquare links in an acre.

PROBLEM I.

To meaſure a three ſided field.

RULE. Meaſure round it, putting down the length of each ſide ſeparately.

EXAMPLE. Suppoſe a field in the following form be to be meaſured.



I meaſure the ſide A B and find it 6.20 (or 6 chains and 20 links,) the ſide B C and find it 3.40, the ſide C A and find it 5.50, and it is done.

To plot it.

Draw a blank line at random as the line AB, then from any ſcale of equal parts take 6.20 with your dividers or compaſſes, and lay it on that line from A to B and draw it with ink; then from the ſcale of equal parts take 5.50, ſet one foot in A and deſcribe an arch, and take 3.40, ſet one foot in B and deſcribe another arch to cut the former arch in C; draw the lines AC and BC and you have the true form of your field.

To find the content.

With your compaſſes take the neareſt diſtance from C to the line A B, and meaſure that on the ſame ſcale that you uſed before, which we will ſuppoſe to be 3.03 for the perpendicular CD; then multiply 3.03 by half 6.20 and the product 9.3930 is the content required; which to reduce into acres, roods, and perches, you

muſt point off 5 figures thus, .93930; multiply
4
by 4, point off 5 figures and ———
the left figure is roods 3.75720; multiply
40
by 40, and point off 5 figures, ———
the left is perches 30.28800.

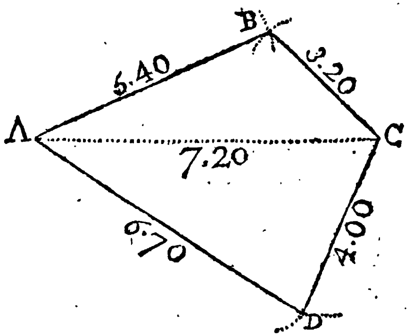
N. B. The reaſon of pointing off 5 figures to the right is the ſame as if the chains and links were ſet down and work'd as one whole number, and the product divided by 100000 (the ſquare links in an acre;) for any number of chains is ſo many hundred links: Otherwiſe if you work the links as the decimal of a chain and divide the product by 10 (the ſquare chains in an acre,) the anſwer in acres, &c. will be the ſame.

PROBLEM II.

To meaſure a four ſided field.

RULE. Meaſure round it, putting down each ſide ſeparately; and alſo meaſure one of the diagonals.

EXAMPLE. Suppoſe a field in the following form to be meaſured.



I meaſure the ſide A B and find it 5.40, the ſide B C and find it 3.20, the ſide C D 4.00, the ſide D A 6.70, and the diagonal A C 7.20, and its done.

To plot the ſame.

Draw the diagonal A C at random, then take 7.20 from any ſcale of equal parts and lay it from A to C; then take 5.40 in your compaſſes from the ſame ſcale, ſet one foot in A and deſcribe an arch; take 3.20 in your compaſſes from the ſame ſcale, ſet one foot in C and croſs the former arch in B; draw the lines A B and B C. Then take 6.70 in your compaſſes from your ſcale and deſcribe an arch, alſo take 4.00 in your compaſſes and croſs the laſt arch in D; draw the lines A D and D C, and ſo you have the true figure of the field.

To find the content.

Now let fall perpendiculars to the neareſt diſtance from the angles B and D upon the diagonal A C, and meaſure theſe perpendiculars by the ſame ſcale of equal parts that you uſed before; and it being a trapezium, you may meaſure it as ſuch; viz. multiply half the diagonal by the ſum of the two perpendiculars, and in the product point off to the right hand 5 figures only (if you have no decimal parts of a link,) and what remains on the left hand will be acres, &c.

See the work in the next page.

Perpen- } 225 ½)720
diculars. } 365

360

590
360

35400
1770

2.12400
4

.49600
40

19.84000

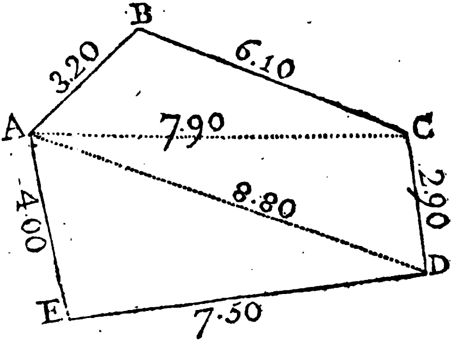
A. R. P.
Content 2 0 19

PROBLEM III.

To meaſure a five-ſided field.

RULE. Meaſure round it, putting down each ſide ſeparately; and alſo meaſure 2 diagonals.

EXAMPLE. Suppoſe a field in the following form to be meaſured.



I meaſure the ſide A B and find it 3.20, B C and find it 6.10, C D 2.90, D E 7.50, and E A 4.00; the diagonal AC I find to be 7.90 and the diagonal A D to be 8.80.

To plot it out.

Draw the diagonal A C at random, and from a ſcale of equal parts take 7.90 and lay it from A to C; take 8.80 from the ſame ſcale, ſet one foot in A, and deſcribe an arch; take 2.90 from the ſcale, ſet one foot in C and croſs the former arch in D; draw the line A D for the ſecond diagonal, and the line C D for one ſide of the field. Then take 3.20 in your compaſſes and deſcribe an arch, and 6.10 and croſs that arch in B; draw the lines A B and B C; take 7.50 in your compaſſes and deſcribe an arch, alſo take 4.00 and croſs that arch in E; draw the lines DE and AE and you have the true figure of your field.

To find the content.

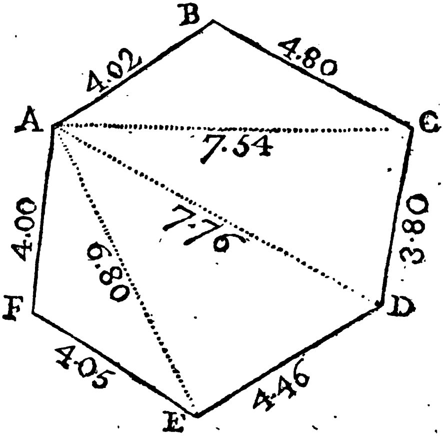
Let fall two perpendiculars on the diagonal A D from the angles C and E, meaſure their lengths from the ſcale of equal parts that you uſed before, and thence find the content of the trapezium A C D E as is before taught. Then let fall a perpendicular from the angle B upon the diagonal A C, find its length from the ſcale of equal parts, which multiply by $\frac{1}{2}$ the diagonal A C and you have the content of the triangle A B C, which added to the content of the trapezium A C D E (before found) will give the content of the whole field as required; remembring to cut off for acres, &c. as before taught.

PROBLEM IV.

To meaſure a ſix ſided field.

RULE. Meaſure round it, and put down each ſide ſeparately; and alſo meaſure 3 of its diagonals.

EXAMPLE. Suppoſe a field in the following form to be meaſured.



I meaſure each ſide and find them the ſame as in the *figure*, and alſo the diagonal A C, A D, and A E, and the field is meaſured.

To plot the ſame proceed as in the laſt example; and for the area you have 4 triangles to find the content of.

1. Let fall a perpendicular from B to the diagonal A C, and meaſure it on your ſcale, which multiply by ½ A C and you have the content of that triangle.

2. Let fall a perpendicular from C to the diagonal A D, and meaſure it on your ſcale, which multiply by ½ AD, and the product is the content of the ſecond triangle.

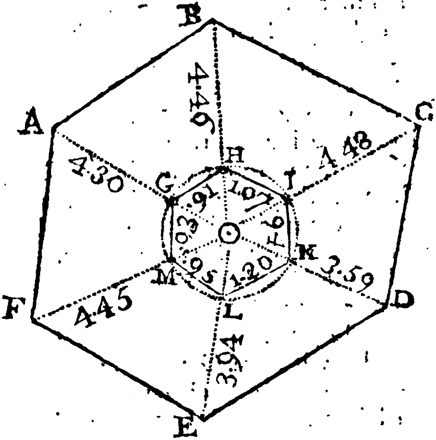
3. Let fall a perpendicular from E on the diagonal A D, and meaſure it on your ſcale, which multiply by ½ A D, and the product is the content of the third triangle.

4. Laſtly let fall a perpendicular from F to the diagonal A E, and meaſure it on your ſcale, which multiply by ½ A E, and the product is the content of the fourth triangle.

5. Add the contents of theſe four triangles together, and the ſum will be the content of the whole field.

PROBLEM V.

To take the plot and content of the former ſix ſided field at one ſtation in any place thereof from whence you may ſee all the angles, as in the following example.



Going into the field chooſe ſome convenient place therein, as at ⊙, and let marks be ſet up at every angle; then take hold of one end of the chain and ſtand in ⊙, and cauſe an aſſiſtant to take hold of the other end, let him go towards the angle A, and when you ſee him exactly in a ſtraight line with the angle A, bid him put down a peg at the end of the chain and leave it there at G; then cauſe your aſſiſtant to go towards the angle B, and when you ſee him in a ſtraight line with the angle B, bid him put down another peg at the end of the chain and leave it there at H; then cauſe him to go towards the angle C, and put down a peg at the end of the chain at I; thus do for all the other angles. And when you are come to M and put down your laſt peg, meaſure from it, to your firſt peg at G, which we will ſuppoſe 93 links; then meaſure from your peg at G to your peg at H and ſuppoſe it to be 91 links; thus do for all the other pegs, and put them down as in the *figure*. Laſtly, meaſure the line ⊙ A and ſuppoſe it to be 4.30; then meaſure the line ⊙ B and ſuppoſe it 4.46, then the line ⊙ C and ſuppoſe it 4.48, and ſo for the lines ⊙ D, ⊙ E, and ⊙ F, and then your work is done in the field; the next thing to be done is to plot it, or make a plan of it, which is done thus.

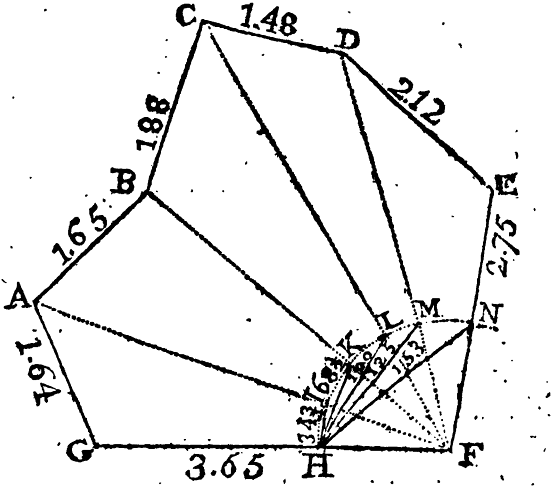
Firſt take the length of 1 chain in your compaſſes from any ſcale of equal parts, and deſcribe a circle; then from the ſame ſcale take 91 links and lay it from G to H; and take 1.07 from the ſame ſcale and lay it from H to I; and ſo for I K 94 links, KL 1.20, LM .95, and MG .93. Then draw lines from ⊙ at random through theſe ſeveral points G, H, I, K, L, M; and from the ſame ſcale of equal parts take 4.30 in your compaſſes and lay it from ⊙ to A; then take 4.46 and lay it from ⊙ to B; do the ſame for the other lines ⊙ C, ⊙ D, ⊙ E, ⊙ F; then draw the boundary lines A B, B C, D E, E F, F A, and thus you have the true figure of your field.

To find the content of this ſurvey, obſerve the rules already laid down; viz. find the content of each ſingle triangle (being here 6) and add their contents together into one ſum, and that will be the content of the whole field.

PROBLEM VI.

To take the plan and content of a field at one ſtation in any angle thereof from whence the other angles may be ſeen, without meaſuring the diagonals.

EXAMPLE. Let A, B, C, D, E, F, G, be the field, and F the angle at which you would take your obſervations.



1. Let an aſſiſtant take the chain and go towards the angle G while you ſtand with the other end of the chain in the angle F; bid him put down a peg at the end of the chain at H, and leave it there.

2. Then cauſe him to go towards the angle A, and when you ſee him in a ſtraight line with A, bid him put down another peg at I, and leave it there alſo.

3. Cauſe him to go towards the angle B, and when you ſee him in a ſtraight line with B, bid him put down a peg at the end of the chain at K; proceed in this manner for the angles C, D, and E.

4. Then go to the peg at H and meaſure to the peg at I which ſuppoſe 34 ¾ links.

5. Then meaſure from H to K which ſuppoſe 68 ½ links.

6. Meaſure from H to L which ſuppoſe 100 links, or 1 chain.

7. Meaſure from H to M which is 125 links.

8. Meaſure from H to N which is 153 links.

9. Meaſure round the field, putting down the meaſure of each particular ſide, and your work is done in the field.

To plan theſe obſervations.

1. Draw the line G F at random, and from any ſcale of equal parts take 100 links, or 1 chain, ſet one foot in F and deſcribe an arch; then take 34 ¾ from your ſcale and lay it from H to I on the arch; and take 68 ½ and lay it from H to K on the arch; alſo, take 100 and lay it from H to L; and 125 and lay it from H to M; and take 153 and lay it from H to N; draw the lines F A, F B, F C, &c. thro' the points I, K, L, &c. at random.

2. From the ſame ſcale of equal parts take 3.65 and lay it from F to G; and take 1.64 in

your compaſſes, ſet one foot in G and croſs the random line F A in A; alſo take 1.65, ſet one foot in A and croſs the random line F B in B; take 1.88 ſet one foot in B and croſs the random line F C in C; proceed in the ſame manner for all the reſt.

3. Draw the lines FG, GA, AB, BC, CD, DE, EF, and you have the true figure of the field.

To find the content.

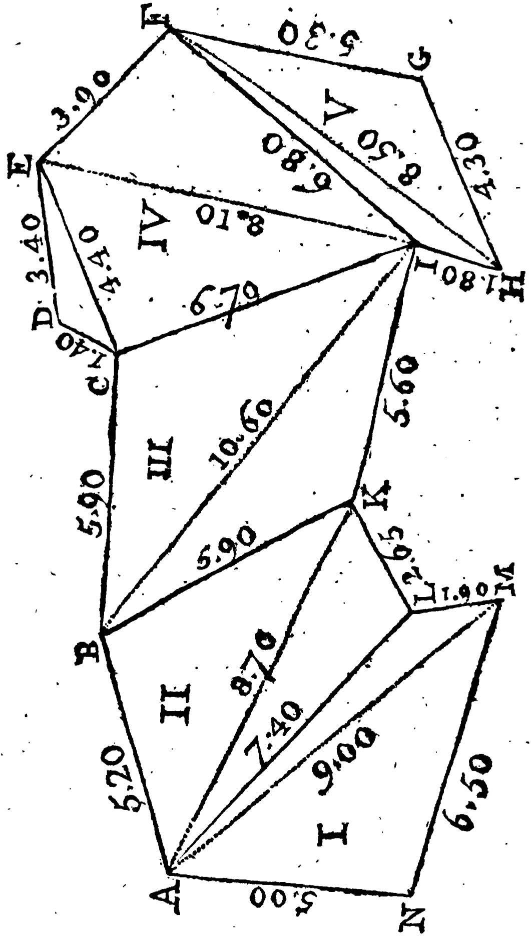
Having meaſured the lines F A, F B, F C, and F D, from the ſame ſcale of equal parts that you took the boundary lines from; let fall perpendiculars as is before taught, and meaſure the 5 triangles as in *problem 4th*, and their ſum will be the area of the field.

N. B. You may as well take the plan of a field at any one ſtation in one ſide thereof as in an angle, the operation being juſt the ſame and therefore we ſhall omit it here.

PROBLEM VII.

To take the plot of a field that is ſo irregular that from no one ſtation you can ſee all the angles.

Let A, B, C, D, E, F, G, &c. be the irregular field to be meaſured.


F
G
5.30
3.90
V
6.80
8.50
4.30
E
IV
8.10
3.40
4.40
I
H
1.80
6.70
D
1.40
C
5.60
III
10.60
5.90
K
5.90
2.65
L
1.90
M
B
II
8.70
7.40
9.00
5.20
6.50
I
A
5.00
N

1. Begin at A and meaſure the trapezium I as is taught in the former problems.

2. Meaſure the trapeziums II, III, IV, V, in like manner; and laſtly meaſure the triangle C D E.

3. Let fall perpendiculars from the angles which ſubtend the diagonals, upon the diagonals, and proceed for the areas of each ſingle trapezium, &c. as is before taught; the ſum of which will be the area of the field.

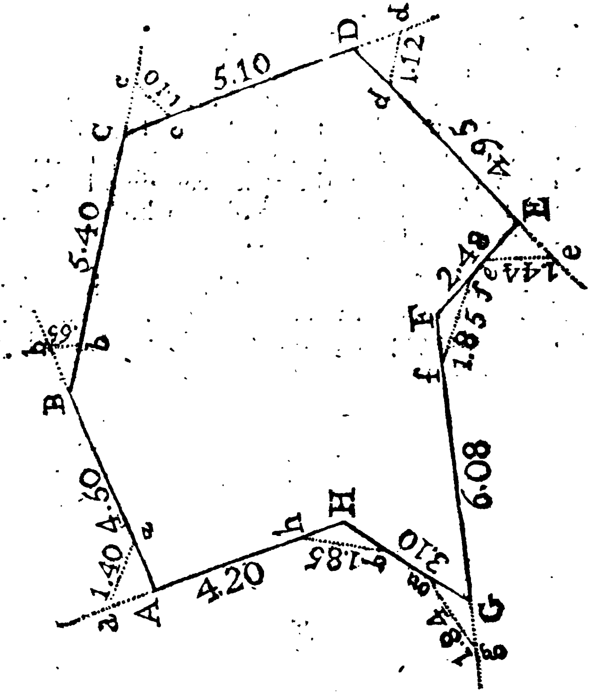
N. B. If an hedge ſhould be crooked, meaſure in a ſtraight line and take offsets at convenient places from that ſtraight line to the hedge; and if you divide the total ſum of ſuch offsets by the number of times taken, it will give a mean breadth for the ſpace contain'd between the ſtraight line and the hedge; which part caſt up as a parallelogram, &c.

Note. If a field be very large and irregular it is much better to meaſure round it and take the angles as you go round, as in the following problem.

PROBLEM VIII.

To take the plot of a wood, common, or a large champian field, by meaſuring round it, and making obſervations at every angle.

EXAMPLE. Let A, B, C, D, E, F, G, H, be a large field or wood through which you cannot ſee to take the angles, but muſt be forced to meaſure round it.



1. Begin at A and meaſure the line A B 4.50; when you are at B let an aſſiſtant take the chain ſtraight forward (yourſelf ſtanding at B,) bid him put down a peg at *b* and leave it there; then meaſure along the line B C one chain, put a peg at *b* and meaſure from b to *b* which ſuppoſe 65 links.

2. Meaſure the line B C 5.40; and when you are at C continue the chain ſtraight forward and put down a peg at the end of it at c; meaſure from C to *c* one chain, put down another peg at the end of the chain, and meaſure the line c *c* which ſuppoſe 1.10.

3. Meaſure the line C D 5.10; and when you are at D meaſure one chain ſtraight forwards and put a peg at d; meaſure from D to *d* one chain, put down a peg at the end of the chain, and meaſure the line d *d* which ſuppoſe 1.12.

4. Meaſure each particular ſide and angle in this manner till you have meaſured all round the field or wood, and then your work is done in the field; and what now remains to be done is to plot it and to find the content.

To plot it.

1. Draw the line A B at random, and from any ſcale of equal parts take 4.50 in your compaſſes and lay it from A to B; take 1 chain in your compaſſes, ſet one foot in B, and mark the line A B continued in b, draw an arch, and lay 65 from b on that arch to *b*; draw a line from B thro' the point *b* at random.

2. Take 5.40 in your compaſſes and lay from B to C; take 1 chain in your compaſſes, ſet one foot in C and mark the line BC continued in c, draw an arch, and lay 1.10 on that arch from c to *c*, draw a line from C through the point *c* at random.

3. Take 5.10 in your compaſſes and lay from C to D; take 1 chain in your compaſſes, ſet one foot in D and mark the line C D continued in d, draw an arch and lay 1.12 on that arch from d to *d*, draw a line from D thro' *d* at random.

4. Proceed juſt in the ſame manner for all the other lines and angles of the field; and if you have done your work right your laſt line H A will juſt come and join with the point A; which if it ſhould go beyond the point A you have laid off ſome of your angles too ſmall; but if it falls ſhort of the point A you have laid off ſome of your angles too large; and that error muſt be corrected accordingly.

To find the content.

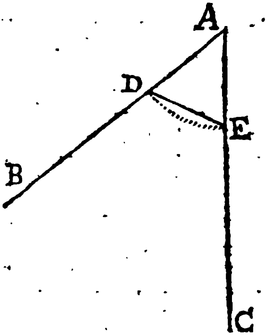
After you have drawn the plan juſtly and truly, you may divide it into trapeziums and triangles, and ſo let fall perpendiculars as is before taught, and thence find the content eaſily.

PRO-

PROBLEM IX.

To take any angle with the chain only.

EXAMPLE. Suppoſe the two ſides of a field were as in the following figure; it is required to find the angle included between them?



Firſt, I ſtand with one end of the chain in the angle A, and lay the chain along the hedge AB, and put down a peg at the end of the chain at D.

Secondly, I lay the chain along the hedge AC, and put down a peg at the end of the chain at E.

Thirdly, Meaſure from the peg at E to the peg at D, and ſuppoſe it meaſures 1.03; then look in the following table for 1.03 and you will find the angle to be 61 deg. 59 min. 42 ſec. ——Or ſuppoſe the extent from D to E was 8[illegible] links; by the table you will find the angle to be 50 deg. 56 min. 6 ſec.; and ſo for any other extent.

The following is a table of the degrees, minutes, and ſeconds, that is contained in any angle made by any number of links; radius being one chain, or 100 *links.*

	0			0 ½			1			1 ½			2			2 ½			3		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
	0	0	0	0	17	11	0	34	23	0	51	34	1	8	45	1	25	57	1	43	8
10	5	43	55	6	1	8	6	18	21	6	35	34	6	52	47	7	10	0	7	27	13
20	11	29	10	11	46	13	12	3	16	12	20	33	12	37	50	12	55	8	13	12	26
30	17	15	14	17	32	37	17	50	1	18	7	25	18	24	50	18	42	15	18	59	40
40	23	4	26	23	21	59	23	39	32	23	57	6	24	14	41	24	32	16	24	49	52
50	28	57	18	29	15	4	29	32	50	20	50	37	30	8	24	30	26	13	30	44	2
60	34	54	55	35	12	56	35	30	58	35	49	2	36	7	6	36	25	12	36	43	18
70	40	58	29	41	16	51	41	35	13	41	53	37	42	12	1	42	30	28	42	48	55
80	47	9	23	47	28	9	47	46	56	48	5	45	48	24	35	48	43	27	49	2	19
90	53	29	15	53	48	16	54	7	48	54	27	7	54	46	27	55	5	50	55	25	14
100	60	0	0	60	19	53	60	39	46	60	59	43	61	19	40	61	39	41	61	59	42
110	66	44	2	67	4	39	67	25	17	67	45	59	68	6	41	68	27	28	68	48	16
120	73	44	23	74	5	55	74	27	28	74	49	6	75	10	44	75	32	29	75	54	14
130	81	5	0	81	27	41	81	50	21	82	13	10	82	35	59	82	58	55	83	21	52
140	88	51	15	89	15	21	89	39	27	90	3	49	90	28	11	90	52	41	91	17	11
150	97	10	51	97	36	57	98	3	3	98	29	19	98	55	42	99	22	13	99	48	50
160	106	15	37	106	44	26	107	13	14	107	42	17	108	11	31	108	40	54	109	10	29
170	116	25	24	116	58	17	117	31	10	118	4	27	118	37	59	119	11	49	119	45	55
180	128	18	58	128	58	52	129	38	46	130	19	27	131	0	39	131	42	53	132	24	41
190	143	36	37	144	33	4	145	29	32	146	28	18	147	28	46	148	31	8	149	35	34

	3 ½			4			4 ½			5			5 ½			6			6 ½		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	2	0	20	2	17	31	2	34	42	2	51	54	3	9	6	3	26	18	3	42	30
10	7	44	27	8	1	41	8	18	55	8	36	9	8	53	23	9	10	38	9	27	52
20	13	29	47	13	47	9	14	4	25	14	21	41	14	39	1	14	56	21	15	13	41
30	19	17	6	19	34	33	19	51	59	20	9	26	20	26	54	20	44	22	21	1	51
40	25	7	29	25	25	6	25	42	48	26	0	21	26	18	1	26	35	41	26	53	20
50	31	1	52	31	19	42	31	37	34	31	55	26	32	13	17	32	31	9	32	49	6
60	37	1	25	37	19	33	37	37	43	37	55	52	38	14	3	38	32	15	38	50	28
70	43	7	24	43	25	52	43	44	23	44	2	55	44	21	29	44	40	3	44	58	39
80	49	21	14	49	40	9	49	59	7	50	18	5	50	37	5	50	56	5	51	15	10
90	55	44	43	56	4	12	56	23	40	56	43	7	57	2	40	57	22	14	57	41	52
100	62	19	47	62	39	52	63	0	2	63	20	11	63	40	25	64	0	39	64	20	57
110	69	9	9	69	30	2	69	50	59	70	11	57	70	33	1	70	54	4	71	15	12
120	76	16	5	76	37	56	76	59	54	77	21	52	77	43	56	78	6	1	78	28	12
130	83	44	58	84	8	3	84	31	19	84	54	36	85	17	55	85	41	13	86	4	45
140	91	41	52	92	6	32	92	31	24	92	56	16	93	21	19	93	46	22	94	11	37
150	100	15	35	100	42	28	101	9	28	101	36	36	102	3	52	102	31	16	102	58	22
160	109	40	13	110	10	30	110	40	18	111	10	36	111	41	8	112	11	51	112	42	46
170	120	20	20	120	55	2	121	30	3	122	5	24	122	41	4	123	17	5	123	53	27
180	133	7	36	133	51	8	134	35	19	135	20	12	136	5	48	136	52	11	137	39	21
190	150	42	18	151	51	37	153	3	50	154	19	22	155	39	43	157	2	35	158	31	50

	7			7 ½			8			8 ½			9			9 ½		
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″
0	4	0	41	4	17	53	4	35	6	4	52	18	5	9	30	5	26	42
10	9	45	7	10	2	22	10	19	38	10	36	54	10	54	9	11	11	39
20	15	31	2	15	48	23	16	5	44	16	23	6	16	40	29	16	57	52
30	21	19	20	21	36	50	21	54	20	22	11	51	22	29	22	22	46	54
40	27	10	59	27	28	41	27	46	23	28	4	6	28	21	49	28	39	34
50	33	7	4	33	25	0	33	42	57	34	0	55	34	18	54	34	36	55
60	39	8	42	39	26	58	39	45	13	40	3	31	40	21	49	40	40	9
70	45	17	15	45	36	3	45	54	52	46	13	24	46	31	55	46	50	39
80	51	34	14	51	53	21	52	12	28	52	31	38	52	50	48	53	10	4
90	58	1	30	58	21	11	58	40	52	59	0	37	59	20	22	59	40	11
100	64	41	16	65	1	45	65	22	13	65	42	35	66	2	58	66	23	30
110	71	36	21	71	57	24	72	19	27	72	40	28	73	1	30	73	22	57
120	78	50	24	79	12	42	79	35	1	79	57	27	80	19	53	80	42	26
130	86	28	16	86	51	56	87	15	37	87	39	26	88	3	16	88	27	15
140	94	36	52	95	2	19	95	27	47	95	53	26	96	19	6	96	44	58
150	103	26	29	103	54	18	104	22	16	104	50	21	105	18	38	105	47	3
160	113	13	54	113	45	15	114	16	13	114	48	36	115	20	38	115	52	54
170	124	30	11	125	7	17	125	44	19	126	22	41	127	1	1	127	39	46
180	138	27	22	139	16	18	140	6	11	140	57	52	141	49	4	142	42	13
190	160	7	38	161	51	45	163	46	51	165	57	24	168	32	11	171	53	44

PROBLEM X.

To meaſure any inacceſſible diſtance by the chain.

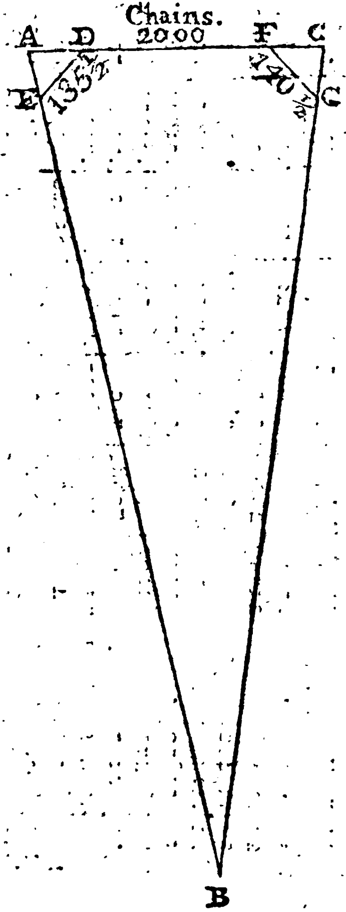
RULE 1. Chooſe two ſtations, and meaſure the diſtance between them; then ſtand at your firſt ſtation, and lay off a chain towards your ſecond ſtation, and leave a peg at the end of the chain; then lay off a chain towards your object, and leave a peg at the end of your chain; meaſure your diſtance between your two pegs, and your work is done at your firſt ſtation. 2. Then meaſure to your ſecond ſtation, and lay one chain towards your firſt ſtation, and leave a peg at the end of the chain; then lay off a chain towards your object and leave a peg at the end of your chain; meaſure the diſtance between your two pegs, and your work is done at your ſecond ſtation. 3. Look in the 1ſt. table for the number of links that was between the pegs at each ſtation, add the two numbers together which you find againſt ſuch numbers in the table, and take the ſupplement of that ſum, that is, ſubtract the ſum from 180 degrees. Then look in the 2d. table which ſhew the natural ſines of the firſt, and ſay thus: As the ſine of the remainder is to the diſtance between the two ſtations, ſo is the ſine of the angle at the firſt ſtation to the diſtance of the object from the ſecond ſtation; and ſo is the ſine of the angle at the ſecond ſtation, to the diſtance of the object from the firſt ſtation. And thus you have two diſtances much more correct than if you had made uſe of the beſt theodolite or plain-table that ever was made.

Next follows a table of factors to find the area of any plain triangle, by having any two ſides and the angle included between them.

	0	0½	1	1½	2	2½	3	3½	4	4½
0	.00000	.00500	.01000	.01500	.01999	.02500	.02999	.03499	.03999	.04498
10	.09987	.10485	.10983	.11479	.11975	.12475	.12972	.13469	.13966	.14462
20	.19913	.20399	.20874	.21375	.21866	.22357	.22847	.23338	.23829	.24317
30	.29661	.30143	.30625	.31107	.31588	.32068	.32548	.33027	.33505	.33983
40	.39192	.39660	.40129	.40596	.41063	.41529	.41995	.42459	.42922	.43387
50	.48412	.48864	.49314	.49763	.50211	.50659	.51105	.51551	.51994	.52437
60	.57236	.57665	.58093	.58520	.58945	.59370	.59793	.60214	.60635	.61054
70	.65573	.65975	.66376	.66775	.67172	.67569	.67964	.68357	.68748	.69139
80	.73321	.73691	.74059	.74426	.74791	.75154	.75515	.75875	.76232	.76588
90	.80373	.80701	.81035	.81363	.81688	.82012	.82334	.82654	.82972	.83287
100	.86602	.86890	.87175	.87455	.87738	.88016	.88290	.88563	.88833	.89101
110	.91868	.92103	.92335	.92565	.92791	.93015	.93235	.93453	.93668	.93879
120	.96000	.96173	.96343	.96510	.96673	.96833	.96989	.97142	.97291	.97436
130	.98791	.98892	.98987	.99079	.99167	.99251	.99330	.95406	.99476	.99543
140	.99980	.99991	.99998	.99999	.99997	.99987	.99975	.99956	.99932	.99903
150	.99216	.99118	.99014	.98904	.98788	.98665	.98537	.98401	.98259	.98109
160	.96000	.95762	.95517	.95264	.95002	.94731	.94452	.94165	.93864	.93562
170	.89549	.89123	.88685	.88234	.87771	.87295	.86807	.86305	.85791	.85521
180	.78460	.77735	.77000	.76240	.75458	.74647	.73832	.72984	.72113	.71216
190	.59327	.57998	.56652	.55235	.53760	.52222	.50614	.48931	.47162	.45300

	5	5½	6	6½	7	7½	8	8½	9	9½
0	.04998	.05498	.05987	.06468	.06995	.07494	.07993	.08492	.08991	.09489
10	.14958	.15453	.15949	.16441	.16938	.17433	.17927	.18421	.18914	.19414
20	.24804	.25292	.25789	.26266	.26753	.27239	.27724	.28209	.28694	.29178
30	.34460	.34936	.35412	.35887	.36361	.36835	.37308	.37780	.38251	.38722
40	.43846	.44307	.44768	.45226	.45709	.46141	.46597	.47052	.47506	.47960
50	.52879	.53319	.53758	.54197	.54636	.55072	.55487	.55941	.56374	.56567
60	.61471	.61888	.62303	.62716	.63128	.63540	.63949	.64355	.64763	.65169
70	.69527	.69914	.70299	.70683	.71065	.71448	.71830	.72204	.72576	.72950
80	.76941	.77293	.77643	.77991	.78337	.78682	.79024	.79364	.79702	.80039
90	.83599	.83909	.84218	.84524	.84828	.85130	.85429	.85726	.86020	.86313
100	.89366	.89628	.89887	.90145	.90399	.90652	.90902	.91147	.91390	.91630
110	.94087	.94274	.94495	.94695	.94889	.95087	.95279	.95463	.95643	.95823
120	.97578	.97716	.97851	.97982	.98109	.98232	.98352	.98468	.98579	.98688
130	.99606	.99664	.99717	.99766	.99810	.99850	.99886	.99916	.99942	.99964
140	.99869	.99829	.99783	.99732	.99676	.99614	.99546	.99472	.99393	.99307
150	.97954	.97791	.97621	.97448	.97261	.97069	.96871	.96665	.96451	.96229
160	.93247	.92923	.92589	.92245	.91892	.91528	.91162	.90770	.90375	.89970
170	.84721	.84166	.83595	.83010	.82410	.81794	.81169	.80512	.79846	.79162
180	.70294	.69344	.68366	.67358	.66329	.65247	.64141	.62980	.61816	.60594
190	.43330	.41212	.39004	.36600	.33998	.31130	.27932	.24266	.19875	.14098

EXAMPLE. Suppoſe I ſee a remarkable object B at a great diſtance, and I would know my diſtance from it.



1. Meaſure the diſtance A C which ſuppoſe 20 chains, lay off a chain from A to D, and alſo from A to E, meaſure the diſtance DE, (which ſuppoſe) 135 ½ links.

2. Go to your 2d. ſtation at C, lay off a chain from C to F, and alſo from C to G, meaſure the diſtance FG, (which ſuppoſe) 140 ½ links.

3. Look in the table for 135 ½ links and you will find it to be

	°	'	"
	85	17	55
look for 140½ & againſt it is	89	15	21
ſum	174	33	16
which ſubtracted from 180 deg. leaves	5	26	44

4. Look in the firſt table for 5 deg. 26 min. 44 ſec. and the neareſt to it is 5 deg. 26 min. 42 ſec. (which is near enough,) and the number of links belonging to it is 9 $\frac{1}{2}$. Then look in the ſecond table for 9 $\frac{1}{2}$, and the number belonging is .09489; look for 140$\frac{1}{2}$ and the number is .99991: Then ſay,

	Ch.		Ch.
As .09489 :	20.00 : :	.99991 : :	210.75,

which is 2 miles, 5 furlongs, and 16 $\frac{1}{2}$ yards, for the diſtance AB.

Again,

	Ch.		Ch.
As .09489 :	20.00 : :	.99664 :	210.06

of 2 M. 5 F. 1 Y. 1 F. for the diſt. B C.

Note. If the number of degrees minutes, &c. in the ſupplement do not fall out very near to the ſame ſought in the table, and you would have your ſine more exact; ſay, As the difference between any two numbers in the firſt table is to the difference of the like ſines in the ſecond table, ſo is what is wanting of the firſt to a number to be added to the ſecond, &c.

PROBLEM XI.

To take a map or plan of any country or kingdom by the chain only.

1. Chooſe ſome remarkable hill or other place where you can ſee a number of towns, villages, pariſh churches, gentlemen's ſeats, mills, &c. and chooſe alſo a ſecond ſtation.

2. Lay

2. Lay off a chain from your place of ſtanding towards each particular object, remembering to leave the pegs ſtanding at each place; but lay one off towards your ſecond ſtation firſt.

3. Meaſure the exact diſtance from one peg to another even to tenths of a link, which write down in your rough sketch, or elſe in your book; and your work is done at your firſt ſtation.

4. Find your diſtance to your ſecond ſtation by *problem* 10*th* (if the land between you and it is very unlevel and mountainous,) but if it is tolerable level and the diſtance not very great meaſure it with the chain.

5. Now you are at your ſecond ſtation lay off a chain from your place of ſtanding towards your firſt ſtation, and alſo towards each particular object juſt in the ſame manner and order that you did at your firſt ſtation; chooſe alſo a third ſtation and meaſure from peg to peg as you did before.

6. Find your diſtance to your third ſtation, and take the new objects which preſent themſelves at the ſecond and third ſtations juſt in the ſame manner as you did at the firſt and ſecond ſtations; proceed in this manner to a fourth, fifth, ſixth, &c. ſtations, till you have been all over the country or kingdom that you would take the plan of. But remember to chooſe your ſtations as far from one one another as you conveniently can, the farther they are from one another the better and more exact your work will be.

To plan your obſervations there are ſeveral methods, but we ſhall mention three, and ſo a perſon may chooſe which he will according as his work requires expedition or exactneſs.

1. METHOD.

Find the diſtance of each particular place or object from your two ſtations by *problem* 10*th*, then from any ſcale of equal parts take the numbers found, for the 1ſt. ſtation, ſet one foot in the 2d. ſtation and deſcribe an arch; take alſo the numbers found for the 2d. ſtation, ſet one foot in the 1ſt. ſtation and croſs the former arch, and that is the place of your object. Proceed in this manner for each of your other objects.

N. B. This is the moſt exact of any method, but it is too troubleſome for practice eſpecially when there are a great number of objects, becauſe two diſtances muſt be found before you can find the place of your object.

2d. METHOD.

Find the angles in the firſt table anſwering to your numbers found by obſervation; lay the center of the ſemicircle to your firſt ſtation and point off thoſe angles; draw lines at random from your firſt ſtation thro' each of theſe points; then do the ſame at your ſecond ſtation as you have done at your firſt, and the places where each correſponding line interſects is the place of that object belonging to thoſe lines.

N. B. This is a very good method, and much more expeditious than the former; tho' without great care you are liable to commit a ſmall error in pointing off the angles by the edge of the ſemicircle.

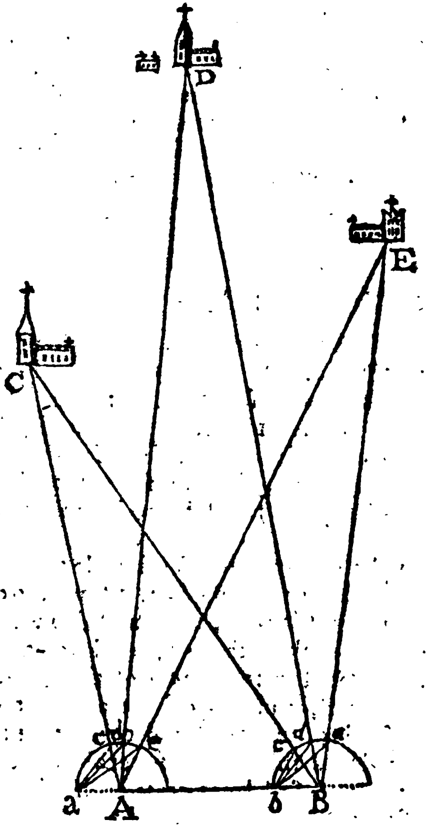
3d. METHOD.

Having laid off the diſtance of your ſtations, take one chain in your compaſſes, ſet one foot in your firſt ſtation and deſcribe a circle, and alſo deſcribe a circle at your ſecond ſtation; then from your ſcale of equal parts take the very ſame numbers that you found by obſervation and lay them on the periphery of the circle, (which if right will end exactly at the very point that you ſet out from) draw lines from your ſtations thro' theſe reſpective points in the circle, and where the correſponding lines interſect, that will be the place of the particular object belonging thereto.

N. B. This is the moſt expeditious of the three methods, but in laying off the points on the periphery of the circle you will be very apt either to fall ſhort of, or go beyond the point that you ſet out from, and that error muſt be corrected by being more careful in pointing off till you come to the very point itſelf that you ſet out from.

To avoid running into this error, inſtead of deſcribing a circle with an extent of one chain only, take from your ſcale the diſtance of 2, 3, 4, 5, 6, &c. chains and deſcribe a circle, and lay on the periphery 2, 3, 4, 5, 6, &c. times the numbers found by obſervation; and if you have a good correct ſcale and ſharp pointed compaſſes, you may draw your plan or map to what degree of correctneſs you pleaſe.

Example. Suppoſe I ſee three towns at an unknown diſtance, and I would know my diſtance from each of them, and alſo the diſtance of the towns from each other.



1. Chooſe two ſtations at a convenient diſtance from each other as A B 64 chains; then ſtand at the ſtation A, and put down a peg at a in a ſtraight line with the ſtation B; lay off a chain towards the town C, and leave a peg at c; lay off a chain towards D and leave a peg at d; lay off a chain towards E and leave a peg at e; meaſure the line ac, ad, and ae. Do the ſame at the ſecond ſtation B, which put down with the angles belonging to them at each ſtation.

STATION A.					STATION B.				
	Links.		°			Links.		°	
ac	126½	or	78	28	*bc*	94½	or	56	24
ad	148¼	or	95	41	*bd*	129	or	80	20
a e	169⅔	or	116	4	*be*	150	or	97	11

N. B. If an external angle is given as a c 78 deg. 28 min. find the internal angle correſponding thereto by ſubtracting it from 180 deg. and the remainder 101 deg. 30 min. is the internal angle required; then ſeek for the natural ſine correſponding, in the ſecond table, and that will be your third term in the ſtating.

2. Lay the center of the ſemicircle to the point A; lay off the angles 78 deg. 28 min, 95 deg. 41 min. and 116 deg. 4 min. and draw lines at random from A thro' thoſe ſeveral points marked off. Then lay the center of the ſemicircle to the point B, and lay off the angles 56 deg. 24 min. 80 deg. 20 min. and 97 deg. 11 min. and draw lines thro' thoſe ſeveral points; and where the two firſt lines croſs each other, that is the ſituation of the town C; and where th

two ſecond lines croſs each other, that is the place of the town D; and where the two third lines croſs each other, that is the place of the town E.

Now the diſtance of each town from your ſtations and alſo the diſtance of the towns from one another may be found by extending from your ſtations to the towns and that extent meaſured on that ſcale of equal parts that your ſtationary diſtance was taken from, will give you the diſtance of any of the towns from either of your ſtations or from one another. But if more exactneſs is required you muſt work by *problem 10th.* and you will find the anſwers as below, which were found by logarithms.

			Ch. lin.		M.	F.	Yds.
Thus the diſtance	from A to	C is	141.89	or	1	6	41 ½
		D is	238.34	or	2	7	183 ½
		E is	196.20	or	2	3	136 ⅖
	from B to	C is	166.91	or	2	0	152
		D is	240.58	or	3	0	12 ¾
		E is	177.63	or	2	1	167 ¾
	from C to	D is	111.06	or	1	3	23 ⅓
		E is	120.48	or	1	4	10 ½
	D to	E is	87.34	or	1	0	161 ½

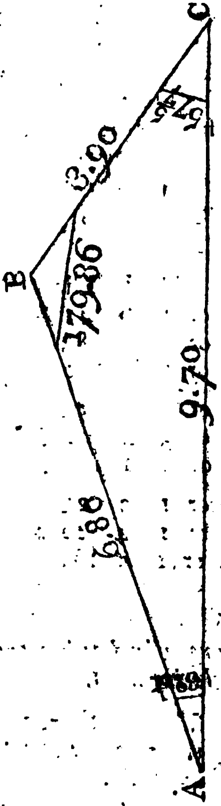
PROBLEM XII.

To find the content of any triangle by having any two ſides and the angle included.

RULE. Multiply the two ſides together, and multiply half the product by the number in the ſecond table anſwering to the angle.

Example. Suppoſe a triangle as in the following figure, what is its area.

N. B. In order to value the fraction of a link in an angle, multiply the difference of any two whole numbers in the table of ſines by the fractional part of a link, and add the product to the leſſer number till you come to 141 $\frac{1}{2}$ links; but ſubtract the value of the difference from the greater number (for the proper factor) if it is greater than 141 $\frac{1}{2}$ links.



AC 9.70
AB 6.80

77600
5820

$\frac{1}{2}$)65.9600

32.9800
.31675

1649000
2308600
1978800
329800
989400

Area 1.0446415000

See the operations of the different angles in the next page.

CA 9.70	BA 6.80
CB 3.90	BC 3.90
87300	61200
2910	2040
½)37.8300	½)26.5200
18.9150	13.2600
.55244	.78654
756600	530400
756600	663000
378300	795600
945750	1060800
945750	928200
1.0449402600	1.0429520400

In each product you'll obſerve one more decimal pointed off than what are in the factors, in order to produce acres, &c. And here we have worked the queſtion three ways, by the 3 different factors belonging to the 3 different angles, and each operation is very nearly the ſame; ſo that it is a matter of indifference which two ſides of a triangle are known (if the angle included is known alſo) to find the area of the triangle; and this is at any time done without having the plan or plot of the triangle.

Now let us find the content of the aforemention'd triangle by the old method when the plan is not given, and then we ſhall ſee how much more troubleſome it is than the new method.

See the work in the next page.

```
      AC  9.70     10.20   10.20   10.20
      AB  6.80      9.70    6.80    3.90
      BC  3.90     ─────   ─────   ─────
         ─────       .50    3.40    6.30
       ½)20.40
         ──────
          10.20
           6.30
         ──────
          30600
         6120
        ───────
        64.2600
           3.40
       ────────
        25704000
       1927800
       ─────────
       218.484000
              .50
       ──────────
     109.24200000(1.04518
     1
     ──
   204)0924
        816
     ──────
   2085)10820
        10425
     ────────
    20901)39500
          20901
     ─────────
   209028)1859900
          1672224
     ────────────
           187676
```

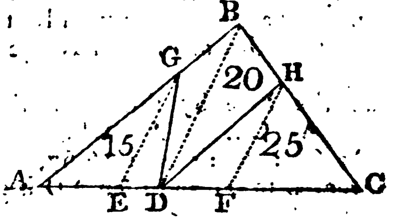
The

The Method of Surveying, *as it relates to* dividing of Land.

PROBLEM XIII.

To divide a triangular piece of land into any number of equal or unequal parts, by lines proceeding from any point aſſigned in any ſide thereof.

EXAMP. Let ABC be the triangular piece of land, containing 60 acres, to be divided between three men; the firſt to have 15 acres, the ſecond 20, and the third 25 acres; and the lines of diviſion to proceed from D.



Firſt meaſure the baſe which is 50 chains; then divide this baſe into 3 parts, thus:

A Ch. A Ch.
As 60 : 50 :: 15 : 12.50 for the firſt man's baſe, which ſet from A to E; then ſay, As 60 : 50 :: 20 : 16.67 for the ſecond

ſecond man's baſe, which ſet from E to F; and the third man's baſe muſt be 20.83, viz. from F to C. This done draw an obſcure line from D to the oppoſite angle B, and from E and F draw the lines E G and F H parallel to B D. Laſtly from D draw D G and D H, which will divide the triangle into three ſuch parts as were required.

PROBLEM XIV.

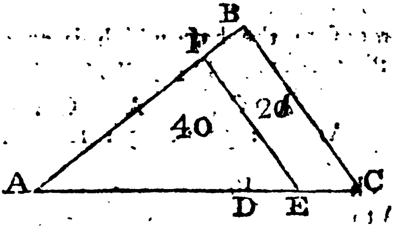
To divide a triangular piece of land according to any proportion given by a line drawn parallel to one of its ſides.

EXAMP. Let ABC be the triangular piece of land containing 60 acres, the baſe A C is 50 chains; this piece of land is to be divided between two men by a line drawn parallel to BC, in ſuch proportion, that one ſhall have 40 acres, the other 20.

1. Divide the baſe by this proportion,

A Ch. A Ch.
As 60 : 50 : : 20 : 16.67,

which lay from C to D; and then AD is 33.33.

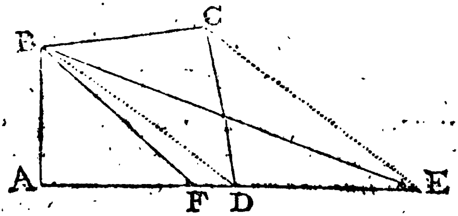


2. Find a mean proportional between A D and A C, viz. 40.82, which ſet from A to E; then draw EF parallel to BC, and the triangle is divided as required.

PROBLEM XV.

To reduce a trapezium into a triangle, by lines drawn from any angle thereof.

Let A B C D be the trapezium to be reduced into a triangle and B the angle aſſigned.



Draw the obſcure line B D, and draw C E parallel to it, produce the baſe A D to E, and draw BE, which will make the triangle ABE equal to the trapezium ABCD.

Now to divide this trapezium according to any aſſigned proportion, is no more than to divide the triangle ABE, as is before taught in *problem* 13*th*. which will alſo divide the trapezium.

EXAMPLE.

Suppoſe the trapezium A B C D contains 124 acres, 3 roods, 8 perches, is to be divided between two men; the firſt to have 50 Ac. 2 R. 3 P. and the other 74 Ac. 1 R. 5 P. and the line of diviſion to proceed from B.

1. Reduce the ſhares into perches, and it will be 8083 for the firſt man, and 11885 for the ſecond man's ſhare.

2. Meaſure the baſe of the triangle, viz. AE 78 chains; then ſay,

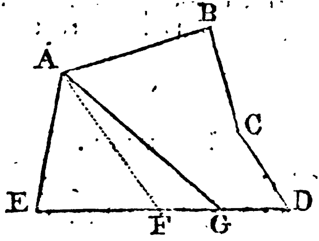
Per. Ch. Per. Ch. l.
As 19968 : 78 : : 8083 : 31.52,

which ſet off from A to F, draw the line BF and the trapezium is divided as required; the triangle ABF being the firſt man's ſhare, and the trapezium BCDF the ſecond's.

PROBLEM XVI.

To divide an irregular plot of any number of ſides according to any proportion given, by a ſtraight line drawn thro' it.

Suppoſe the field ABCDE contains 46 acres to be divided into two equal parts between two men, by a line proceeding from A.



1. Draw a line at pleaſure thro' the *figure*, as AF; then caſt up the content of either half, and ſee what it wants, or what it is more than the true half ſhould be.

2. Thus I caſt up the content of AEF, and find it to be but 15 acres; whereas the true half is 23 acres; 8 acres being in the part ABCDF more than in AEF; therefore I make a triangle containing 8 acres, and add it to AEF, as the triangle AGF; then the line AG parts the *figure* into two equal parts.

Thus you may divide any piece of land of never ſo many ſides and angles, according to any proportion, by ſtraight lines drawn thro' it, with as much certainty, and more expeditiouſly than by any other way yet known.

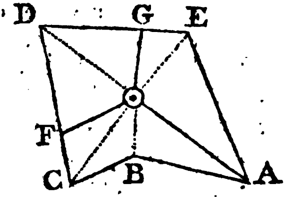
Another EXAMPLE will make all plain.

Suppoſe the following field, containing 27 acres, is to be divided between 3 men, each to have 9 acres, and the lines of diviſion to run from a pond in the field ſo that each may have the benefit of the water, without going over one another's land.

1. From the pond ⊙ draw lines to every angle, as ⊙A, ⊙B, ⊙C, ⊙D, ⊙E; and then the *figure* is divided into 5 triangles, each of which meaſure, and put down the contents ſeverally; which contents reduced into perches will ſtand thus,

A⊙B = 674
B⊙C = 390
C⊙D = 1238
D⊙E = 911
E⊙A = 1107
Perches.

4320



The whole content being 4320 perches or 27 acres, each man's proportion being 1440 perches.

From ⊙ to any angle draw a line for the diviſion line, as ⊙A; then conſider that the firſt triangle A ⊙ B is but 674 perches, and the ſecond triangle B ⊙ C 390, both together make but 1064 perches, which is leſs than 1440 by 376. You muſt therefore cut off from the third angle C⊙D 376 perches for the firſt man's dividing line, which you muſt find out thus: The baſe DC is 18 chains, the content of the triangle 1238; then ſay,

P. C. P. C.
As 1238 : 18 :: 376 : 5.45

which ſet from C to F, and drawing ⊙F, you have the firſt man's part, viz. A⊙F.

Then ſee what remains of the triangle ⊙D, 376 being taken out you will find it to be 862 which is leſs than 1440 by 578: therefore from the triangle D⊙E cut off 578 perches, and the point of diviſion will fall in G; draw the line ⊙G, which with ⊙A and ⊙F divides the *figure* into three equal parts.

PROBLEM XVII.

To find what point of the compaſs any hedge or other object is upon, by the chain.

RULE. 1. Lay off one chain along the hedge (leaving a peg there) and another ſtraight towards the ſun; take the diſtance between the two pegs, which look for in the firſt table, and take the angle belonging thereto. 2. Take the ſun's altitude at that time with a quadrant, and alſo the hour and minute of the day, which being reduced into degrees and minutes of the equator, ſay thus,

As ſine of the hour from noon is to the complement of the ſun's altitude, ſo is the co-ſine of the ſun's declination to the ſine of the ſun's azimuth, which ſubtracted from 180 degrees gives the ſun's azimuth from the ſouth; and the former angle between the ſun and hedge being added or ſubtracted (as occaſion ſhall require) will give the true point of the compaſs that the hedge is upon.

QUEST. Suppoſe on April 10, at 18 minutes paſt 9 in the morning I obſerve the ſun to be 33 deg. 50 min. high, and the angle between the hedge and ſun to be 42 links, which is 24 deg. 14 min. 41 ſec. and the hedge lies more remote from the ſouth than the ſun; I demand what point of the compaſs it is upon?

Here we have given the hour from noon 40 deg. 30 min. ſun's altitude, 33 deg. 50 min. ſun's declination north 8 deg. 3 min.

	°	'
As co-ſine * of the ſun's altitude	56	10
To ſine of the hour from noon -	40	30
So is co-ſine of the ſun's declination	81	57
To ſine of ſun's azimuth from ſouth	50	44

To which add the angle which belongs to 42 links, and the ſum is 74 deg. 58 min. 41 ſec. from the ſouth toward the eaſt, viz. ESE ¾ E fere.

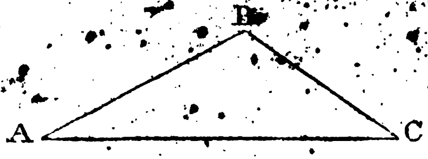
N. B. The ſun's azimuth may be found by various other methods, which the ſurveyor may uſe as he ſees proper.

PROBLEM XVIII.

To take the horizontal line of an hill.

When you meaſure an hill you muſt meaſure the ſuperficies thereof, and accordingly caſt up the content; but when you plot it down, (becauſe you cannot make a convex ſuperficies on the paper) you muſt only plot the horizontal line or baſe thereof, which you muſt ſhadow over with the reſemblance of an hill. That horizontal line is found in this manner.

Suppoſe A B C be an hill whoſe baſe you would know.



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* Co-ſine or ſine complement is what the angle wants of 90 degrees.

Having a quadrant with you, ſtand at A, cauſe a mark to be ſet up at B, ſo high above the top of the hill, as you hold the quadrant from the ground at A; then take the angle A 28 deg. 30 min. meaſure the diſtance AB 24 chains; then go to C and take the angle C 39 degrees, ſubtract the ſum of theſe two angles from 180 degrees and there remains 112 deg. 30 min. for the angle B; then ſay,

As ſine C ; AB : : ſine B : AC ; that is
Ch. Ch. l.
As ſine 39 ; 24 : : ſine 112 30 : 35.23 AC.

N. B. The above may be done by the chain only, but it is a very prolix method; therefore we need not abuſe the chain by aſcribing methods thereto which may be eaſier attained by other means; and if there be any greater improvement capable of being made by the chain than what I have pointed out, I ſhall be more than amply ſatisfied if I have contributed any thing towards it.

Of

OF SPECIFIC GRAVITY.

SPecific gravity is the comparative weight of bodies of equal bulk. Thus if a cubic inch of fine copper be 9 times heavier than a cubic inch of water, we ſay the ſpecific gravity of copper is 9, or its weight is 9 times greater than that of water.

The ſpecific gravity of fluids are readily determined by weighing one and the ſame ſolid in them ſeverally; for ſince we ſuppoſe the ballance in equilibrio with the body ſuſpended in the air, the equilibrium muſt be deſtroyed when the ſolid is immerſed in the fluid, and muſt be then reſtored by weights put into that ſcale to which the body is appended; theſe weights will expreſs the gravity of an equal bulk of the reſpective fluids, and conſequently may be thus compared with each other, or all of them with the gravity of common water as uſual, and diſpoſed in a proper table, making that of water 1000.

In the ſame manner, if divers ſolids are firſt weighed in air, and then immerſed in the ſame fluid, as water, for inſtance, the equilibrium will be deſtroyed, which will be reſtored as before by putting in ſo much weight as is equal to the weight of the ſame bulk of water: The gravity therefore of every ſolid is thus compared with water, and conſequently with each other as in the following table.

Note. The application of this doctrine of ſpecific gravity is not only to compare the weight of a magnitude of one kind, in relation to another kind of body, tho' this is of great uſe in computing the weights of ſuch bodies as are too heavy or too unwieldly to have their weights diſcovered by other means, but 'tis of the utmoſt uſe in diſcovering the goodneſs of any kind of foſſils, drugs, metals, precious ſtones, &c. whether they are genuine or counterfeits; hence appears the uſefulneſs it is of to phyſicians, chemiſts, apothecaries, jewellers, goldſmiths, farmers, &c. the exact knowledge of which muſt conduce to their intereſt. But the great benefit it will be of to the ſeveral uſes of life in making theſe matters familiar will be evident from the following inſtances: Having firſt premiſed that a cubic foot of common water weighs very exactly 1000 ounces avoirdupois weight, or 62 pounds and an half; which may be very readily reduced to troy weight, by a conſideration of the rules laid down at the end of reduction.

A Table of metals and other bodies, ſhewing their ſpecific gravities to rain water; as alſo the number of ounces in a ſolid foot, avoirdupois weight.

Bodies.	*Spec. grav.*	*Bodies.*	*Spec. grav.*
Air - - -	1.2	Bees-wax, white	865
Ale - -	1028	Ditto, yellow	960
Aqua fortis -	1300	Biſmuth - -	9700
Aqua regia -	1234	Blue-ſlate -	3500
Aſh, dry -	838	Blue-ſtone -	2740
Barley laſt year	658	Box, dry - -	1030
Beech, dry -	700	Braſs, caſt - -	8104
Bees-wax -	995	Brandy - -	927

Bodies.	*Spec. grav.*	*Bodies.*	*Spec. grav.*
Brazil	1031	Lead, oar	6800
Brick	2000	Ditto, worſe	6200
Brimſtone	1800	Lignum vitæ	1327
Butter	940	Linſeed oil	923
Cedar	613	Logwood	913
Chalk	2370	Magnet or Loadſtone	5106
Clay	1718		
Coal	1255	Mahogany	1063
Copper	8795	Man's body	1111
Ditto, fine	9000	Marble	2707
Ditto, oar	5775	Milk	1030
Cork	240	Oak ſappy, 1 year old	870
Crabtree, dry	700		
Diamond	3517	Heart of do. dry	925
Earth	1984	Paving-ſtone	2460
Ebony	1177	Pitch	1190
Elm, new	800	Quickſilver	13762
Ditto, dry	600	Rain water	1000
Fir, yellow, dry	657	River water	1009
Do. white, dry	569	Sea water	1030
Flint-ſtone	2582	Silver, caſt	10528
Free-ſtone	2352	Ditto, fine	11091
Glaſs	2600	Do. Engl. coin	10629
Gold, caſt, or ſtandard	18888	Spelter	7065
		Spts. of Turpen.	874
Do. fine, pure	19640	Steel	7850
Horn	1832	Sulpher, com. vide Brimſt.	1800
Ice	908		
Iron, caſt	7135	Sulpher-vivum	2000
Ditto, bar.	7764	Tin	7550
Iron-ſtone	3600	Ditto, oar	480
Ivory	1832	Wheat	757
Lead, fine	11313	Ditto meal	495
Ditto, common	10130	Yew	760

N. B. If the ſpecific gravity of any ſolid in the table be leſs than 1000 it will ſwim in water, but if greater than 1000 it will ſink.

As water is made the ſtandard for ſpecific gravity, its gravity is repreſented by unity or 1, or 1000 where three cyphers are added to give room to expreſs the gravities of other bodies in larger numbers in the table; in doing this, we have a twofold advantage, the firſt is, that by this means we can expreſs the ſpecific gravities of bodies to a much greater degree of accuracy and exactneſs; the ſecond is, that the numbers in the table do alſo expreſs the avoirdupois ounces contained in a cubic foot of every ſort of matter therein ſpecified, becauſe a cubic foot of common water is found by experiment to weigh very nicely 1000 ounces.

Suppoſe I would know how many times any quantity of glaſs is heavier than the ſame quantity of water; then I find in the table againſt glaſs 2600, which if the point of unity was put would be 2.600, the meaning of which is that glaſs is $2\frac{6}{10}$ times heavier than the ſame quantity of water.

Now in order to find what relation a ſolid foot or 1000 ounces has to a ſolid inch, let us conſider that 1728 ſolid inches is 1000 ounces, and conſequently,

In. oz. In.

As 1728 : 1000 : : 1 : .5787

of an ounce; ſo that a ſolid foot of water is to a ſolid inch of water as 1000 to .5787, which divided by 16 gives .036169 for the weight of a ſolid inch of water in the decimal parts of a pound.

Alſo when you have gold, ſilver, electuaries, &c. which is generally weighed by troy weight, you muſt firſt reduce it into avoirdupois weight to agree with the foregoing table.

CASE I. *The dimenſions or ſolidity being given, to find the weight.*

RULE. Multiply the ſolidity in inches by the ſpecific gravity, and divide the product by 1728 (the ſolid inches in a foot,) and the quotient will be the weight in ounces avoirdupois.

If an oak beam be 120 inches long, and 11 by 9, how heavy is it?

```
       11
        9
     ----
       99
      120
    -----
     1980
     99
   ------
    11880
      925
   ------
    59400
   23760
  106920
 --------
 10989000
```

See the remainder of the work in the next page.

Ounces. lb. oz. dra.
1728)10989000(6359.37 or 397 7 6
10368 Anſwer.

6210
5184

10260
8640

16200
15552

6480
5184

12960
12096

864

Note. As the ſpecific gravity of dry heart of oak is different from that which is ſappy, when you meet with oak that is part heart and part ſap, you muſt take ſuch proportion between theſe two numbers that there is between the heart and ſappy part; obſerve the ſame alſo with regard to the moiſture or dryneſs thereof.

Suppoſe an iron ball be 8 inches in diameter, what is its weight?

```
                         8
                         8
                      ----
                        64
                         8
                      ----
                       512
                     .5236
                  --------
                      3072
                     1536
                    1024
                   2560
                 ---------
                 268.0832
                     7135
                 ----------
                 13404160
                8042496
               2680832
             18765824
             ---------------  Ounces.    lb. oz. dra.
      1728)1912773.6320(1106.93  or  69   2   14
            1728              Anſwer.
            -----
             1847
             1728
            ------
              11973
              10368
            -------
               16056
               15552
              -------
                 5043
                 5184
                -------
```

Suppoſe I have a large grinding ſtone for a blade mill, whoſe diameter is 70 inches; and thickneſs 15 inches; what will the carriage come to at 3 d. per hundred weight?

```
          70
          70
       -----
        4900
       .7854
       -----
       19600
      24500
     39200
    34300
    ---------
   3848.4600
          15
    ---------
   192423000
   38484600
    ---------
  57726.9000
     2352
  ----------
    1154538
   2886345
  1731807
 1154538
 -----------
 135773668.8
```

See the remainder of the work in the two following pages.

Ounces.

1728)135773668.8(78572.72

12096

14813

13824

9896

8640

12566

12096

4708

3456

12528

12096

4320

3456

864

lb.

16(78572.72(4910.795

64

145

144

17

16

127

112

152

144

80

80

See the remainder of the work in the next page.

As

```
   lb.   d.    lb.
As 112 : 3 :: 4910.795
                     3
          ------------ Pence     s. d.
      112)14732.385(131.539 or 10 11½.156
          112          Anſwer.
          ---
          353
          336
          ---
           172
           112
           ---
            603
            560
            ---
             438
             336
             ---
             1025
             1008
             ----
               17
```

This is the method for bodies that are ſolid throughout; but for hollow bodies, one method is, to find the ſolidity of its greateſt bulk and ſubtract the concavity therefrom, and proceed with the remainder as in the laſt example; but the following will make all clear, being done a nearer way.

If an iron bomb ſhell be 3 inches thick, and greateſt diameter 14 inches, what is its weight?

14 greateſt diameter.
6 twice the thickneſs.
8 leaſt diameter.
8
64
8
512

14
14
56
14
196
14
784
196
2744
512
2232
.5236
13392
6696
4464
11160
1168.6752
7135
58433760
35060256
11686752
81807264
8338497.5520

See the remainder of the work in the next page.

```
               Ounces.     lb.  oz.
1728)8338497(4825   or    301   9
     6912        Anſwer.
     -----
     14264
     13824
     -----
       4409
       3456
       ----
        9537
        8640
        ----
         897
```

Case II. *The weight being given, to find the ſolid content.*

Rule. Divide the weight in ounces by the ſpecific gravity, the quotient multiplied by 1728 will be the ſolidity in inches.

Suppoſe an iron ſhot weighs 49 lb. how many ſolid inches doth it contain? And alſo what is its diameter?

See the work in the next page.

49

16

294

49

7135)784.0(.10988

7135

70500

64215

62850

57080

57700

57080

620

Inches.

362.629(7.13

343 Diameter.

14911)19629

14911

1518699)4718000

4556097

161903

.10988

1728

87904

21976

76916

10988

189.87264

Solid inches.

.5236)189.87264(362.629

15708

32792

31416

13766

10472

32944

31416

15280

10472

48080

47124

956

How may ſolid inches are there in a marble ſtatue that is 280lb.?

280
16

1680
280

2707)4480(1.655
2707

17730
16242

14880
13535

13450
13535

1.655
1728

13240
3310
11585
1655

Anſ. 2859.840

Note. In like manner the ſolid inches in a cup, tankard, glaſs bottle, &c. may be found, by making uſe of the proper ſpecific gravities, and working as above.

What muſt be the thickneſs of that lead, that one ſquare foot may weigh 6 lb.?

6
16

10130)96.000(.009476
91170

48300
40520

77800
70910

68900
60780

8120

.009476
1728

75808
18952
66332
9476

144)16.374528(.11371 of an inch. Anſ.
144

197
144

534
432

1025
1008

172

And in this manner you may find the thickneſs of any lead pipe, or other thing made of different materials, whoſe weight and ſuperficial content are known; for if they are hollow cylindrical bodies, you muſt multiply the circumference by the length and that gives the ſuperficies, and work with that as in the above example.

How many ſolid inches are there in a man's body that weighs 150 lb.?

```
   150                  1728
    16                  2.16
  ----                 -----
   900                 10368
  150                  1728
  ----                3456
1111)2400(2.16         -------
     2222              3732.48
     ----              Anſwer.
      1780
      1111
      ----
       6690
       6666
       ----
         24
```

How many ſolid inches are there in a ſilver tankard that weighs 23 ounces (avoirdupois)?

```
10528)23.000(.002184        .002184
      21056                    1728
      -----                 -------
      19440                   17472
      10528                  4368
      -----                15288
       89120               2184
       84224               ---------
       -----          Anſ. 3.773952
        48960
        42112
        -----
         6848
```

CASE

CASE III. *The weight and magnitude being given, to find the ſpecific gravity.*

RULE. Divide the weight in ounces by the ſolidity in cubic feet, the quotient will be the ſpecific gravity.

If a piece of marble contains 4 ſolid feet, and weighs 675lb. what is its ſpecific gravity?

```
      675
       16
     ----
     4050
     675
     ----
  4)19800
     ----
Anſ. 2700
```

I have a piece of timber that contains 6 feet, and weighs 285lb. what wood is it?

```
     285
      16
    ----
    1710
    285
    ----
  6)4560
    ----
```

760 ſpecific gravity; then look in the table of ſpecific gravities for that number, and the name of the wood againſt it is yew. But you muſt obſerve that the ſpecific gravity of any ſort of wood varies in the ſeaſoning; and other commodities vary according to their quality or goodneſs.

N. B. All bodies of what nature or kind ſoever being weighed in open air and ballanc'd by thoſe whoſe ſpecific gravity is greateſt, I ſay theſe bodies whoſe ſpecific gravity is leaſt will weigh the heavieſt in vacuo.

Thus if a piece of lead at the end of a nice ballance and a piece of cork at the other end are in equilibrio in the air, and thus placed under the receiver of an air-pump, as ſoon as the air begins to be exhauſted, the equilibrium will begin to be deſtroyed, till at laſt when all the air is taken away, the cork will deſcend and ſhew itſelf really heavier than the lead. The reaſon of which is very evident from the laws of hydroſtatics; for both bodies, being weighed in air, each would loſe the weight of an equal bulk of air, conſequently the cork will loſe a greater weight than the lead, becauſe it is of greater bulk; and therefore when the air is taken away from both, the weight that is reſtored to the cork being greater than what is reſtored to the lead, will cauſe it to preponderate or weigh down the lead in vacuo.

Now let us take 14lb. of cork, which is 224 ounces, and $\frac{224}{240}$ is equal to .93 of a foot, which multiplied by 1728 gives 1612.8 ſolid inches in 14lb. of cork: And $\frac{224}{10130}$ is equal to .022112 of a ſolid foot, which multiplied by 1728 gives 38.2996 ſolid inches in 224 ounces of lead: Then 38.2096 ſubtracted from 1612.8 leaves 1574.5904 for the ſolid inches of air to be added to the cork; the weight of which quantity of air is thus found; As 1728 : 1.2 :: 1574.59 to 1.09346, equal to 1 oz. 1 $\frac{1}{2}$ drm, therefore the 14lb. of cork is 1 oz. 1 $\frac{1}{2}$ drm. heavier than the 14lb. of lead.——And for the ſame reaſon a pound of feathers is heavier than a pound of lead; which may ſeem a paradox to ſome; but in reality is no greater a paradox than the following one in finding the center of gravity, *viz.* That if a tapering piece of timber be nicely ballanced, and then cut in two at the place where the ſupport for the ballance is, I ſay that if one piece is put into one ſcale and the other piece into the other ſcale, they will not ballance each other as they did before, but the tapering end will be the lighteſt.

In the foregoing example we make the cork and the lead to ballance each other in the fluid medium of air wherein neither could ſwim; now in the following example we will chooſe two bodies that will not ſwim in water, and then ſee how much the lighteſt body is heavier than the heavieſt in air: And for this purpoſe we ſhall chooſe brazil-wood whoſe ſpecific gravity is 1031, and lead whoſe ſpecific gravity is 10130.

Now ſuppoſe we take 15lb. of each, that is 240 ounces, then 240 divided by 1031 gives .2327 part of a ſolid foot, which multiplied by 1728 gives 402.1 ſolid inches in 15lb. of brazil-wood: Again, 240 divided by 10130 gives .02369 part of a ſolid foot, which multiplied by 1728 gives 40.936 ſolid inches in 15lb. of lead. Now theſe different bulks muſt remove different bulks of water, and conſequently that of the greateſt bulk will meet with the greateſt reſiſtance from the water as it endeavours to ſink, and therefore the equilibrium will be deſtroyed; now to find how much the equilibrium will be deſtroyed let us find the difference of their bulks, which is 361.164 ſolid inches equal to the quantity of water diſplaced by the brazil-wood more than is diſplaced by the lead. Now the weight of the water is found thus; As 1728 : 1000 : : 361.164 : 13lb. 1 oz. for the difference of their weights in water; that is, the brazil-wood would require 13lb. 1 oz. to be added to it, to make a ballance with the lead; conſequently if the ballance was firſt made in water and then weighed in air afterwards, the 13lb. 1 oz muſt have been added to the lead to make it ballance the brazil-wood.

CASE IV. *To determine the goodneſs of metals, &c.*

RULE. 1ſt. Multiply the difference of the ſpecific gravities of the compound and lighter ingredient, by the product of the ſpecific gravity of the heavieſt ingredient and abſolute weight of the compound, for a dividend; 2d. Multiply the difference of the ſpecific gravities of the ingredients by the ſpecific gravity of the compound, for a diviſor; 3d. Divide the dividend by the diviſor, and the quotient will be the weight of the heavieſt ingredient in ounces.

I have a piece of metal whoſe ſpecific gravity is 15840, abſolute gravity (or weight) 80 ounces (avoirdupois;) what is the quantity of gold, and alſo the quantity of ſilver contained therein?

See the work in the next page.

```
        18888                15840
        10528                10528
      ---------            ---------
         8360                 5312
        15840
      ---------
       334400
       66880
      41800                  18888
      8360                      80
   -----------             ---------
    132422400               1511040
                               5312
                           ---------
                            3022080
                           1511040
                          4533220
                         7555200
                      -------------
            132422400)8026644480(60.6139
                       794534400
                      -------------
                          813004800
                          794534400
                         -----------
                          184704000
                          132422400
                         -----------
                           522816000
                           397267200
                          -----------
                           1255488000
                           1191801600
                          ------------
                             63686400

   Silver 60.172 }
   Gold   19.828 } Answ.
         ---------
Compound  80.000
         ---------
```

I have 140 oz. of a mixture of copper and ſilver, whoſe ſpecific gravity is 9640; what quantity of ſilver is there in it?

```
    10528        9640             10528
     8796        8796               140
    -----        ----           -------
     1732         844            421120
     9640                        10528
    -----                       -------
    69280                       1473920
   10392                            844
  15588                         -------
 --------                       5895680
 16696480                      5895680
                              11791360
                              ---------
                16696480)1243988480(74.5
                         116875360
                         ---------
                          75234880
                          66785920
                          --------
                          84489600
                          83482400
                          --------
                           1007200
```

Anſw. {74.5 copper.
{65.5 ſilver.

140.0

Quere, whether the aſſaying of gold, ſilver, &c. would not be much better performed upon hydroſtatical principles than by the preſent method by fire?

Note. That in buying a quantity of gold, ſilver, or any thing which is heavier than braſs (viz. the weights being of braſs,) we ſhould chooſe the lighteſt air, (i. e.) when the mercury in the barometer is loweſt; but in buying precious ſtones, pearls, or any thing that is lighter than braſs, the beſt time to do it is when the air is heavieſt and moſt buoyant, viz. when the quickſilver ſtands higheſt in the barometer: But in ſelling gold or jewels, the contrary rules are to be obſerved in regard to the gravity of the air.

CASE V. *The ſolidity of any timber being given, to find how far it will ſink.*

RULE. Divide the ſpecific gravity of the timber by the ſpecific gravity of the water, multiply the quotient by the depth of the timber, and that gives the inches under water.

How many inches will a cubic foot of aſh or deal ſink in common water?

	Aſh.	Deal.
	1000)838.000(.838	1000)657(.657
	12	12
	10.056 inches.	7.884 under } water.
	Anſwer.	4.116 above }

CASE VI. *The ſolidity of any timber being given, to find how much it will carry.*

RULE. Subtract the ſpecific gravity of the timber from the ſpecific gravity of the water, the remainder is the number of ounces that one ſolid foot will carry.

How much weight is juſt neceſſary to ſink a cubic foot of deal in ſea water?

See the work in the next page.

1030
657
—— lb.
16)373(23.313
32 Answer.
——
53
48
——
50
48
——
20
16
——
40
48
——

How much weight will a raſt made of 12 pieces of yellow deal carry (in ſea-water) if each piece is a foot ſquare and 20 feet long?

1030
657
——
373
240
——
14920
746
—— lb.
16)89520(5595 will juſt ſink the deal under water, and keep the weight compleatly above water.
80
——
95
80
——
152
144
——
80
80
——

12
20
——
240

Now ſuppoſe it was loaden with braſs, whoſe ſpecific gravity 8104 being divided by 16, gives 506.5 pounds; and the pounds in a ſolid foot of ſea water is 64.4 pounds.

```
            5595
             64.4
          -------
           22380
          22380
         33570
       ---------
506.5)360318.0 (711.3
        35455    5595
       -------  ------
          5768   6306.3  viz. it will prevent
          5065             6306.3 lb. of braſs
        -------            from ſinking, and
           7030            will juſt keep it ſuſ-
           5065            pended in the water.
         -------
           19650
           15195
         -------
            4455
```

In the ſame manner is found by the comparative weight of the bodies, how much of any heavy quality or matter may be ſupported by the water.

How many inches will an aſh beam ſink in common water that is 14 by 11, and 18 feet long, when it has 200 lb. upon it?

```
      14
      11
    ----
     154
      18
    ----
    1232
     154
    ----
144)2772(19.25 ſolid feet.
     144
    ----              1000
    1332               838
    1296             -----
    ----               162 ounces that one
     360                ſolid foot will carry.
     288
    ----
     720
     720
    ----

           19.25
            162
          ------
           3850
          11550
          1925
          ------
          3118.50
```

See the remainder of the work in the next page.

lb.
16)3118.50(194.9 that is, it will but juſt carry 194.9 lb. and 195 lb. will ſink it.
16
———
151
144
———
78
64
———
145
144
———
1—

Now ſeeing that it will not carry 200 lb. as the queſtion requires, let us ſuppoſe that 190 lb. was put upon it, then the operation will be as follows:

lb. In. lb.
As 62.5 : 1728 : : 190
190
———
155520
1728
——— In.
62.5)328320.0(5253 which divided by the ſquare inches of the ſurface gives the anſwer requir'd: See the work to each ſide uppermoſt.
3125
———
1582
1250
———
3320
3125
———
1950
1875
———
75

Broadeſt ſide uppermoſt.

28

12

216

14

864

216

Surface 3024)5253(1.737

3024

22290

21168

11220

9072

21480

21168

312

1000)838(.838

11

Inches 9.218 that it ſinks naturally.

1.737 ſinks with 190lb. upon it.

10.955 ſinks in all ; ſo that there

is but .045 out of water.

11.000 depth.

Narroweſt ſide uppermoſt.

18
12
216
11
Surface 2376)5253(2.21
4752
5010
4752
2580
2376
204

1000)838(.838
14
3352
838
11.732 ſinks naturally.
2.210 ſinks more with the weight.
13.942 ſinks in all.
.058 out of the water.
14.000 depth.

We might now proceed to ſhew the principles upon which bodies that are heavier than water are made to ſwim; but as this part of the hydroſtatics is upon a principle different from that of ſpecific gravity, we ſhall omit ſaying any thing on that head, unleſs it ſhould be deſired in a ſecond edition.

AN

AN INTRODUCTION TO PRACTICAL MUSIC.

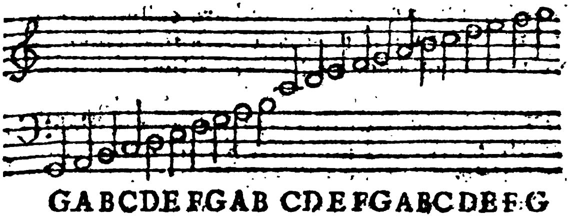
IN practical muſic you'll obſerve there are but ſeven degrees of tones in the gamut, C D E F G A B; and if we proceed further it can be but by repeating the firſt note, and ſo on, ſo that the ſame notes repeated are called octaves: Thus the two notes that create the octave, are in main but one, and ſerve as limits or bounds to all the intervals, ſince all the notes in the gamut are included in an octave; 5 of which are whole tones, and the other 2 half tones, conſequently making 12 half tones in the whole.

But the writers on ſpeculative muſic, not taking the ear to be a competent judge in a caſe ſo nice, have choſen to diſtinguiſh the 12 half notes, not by equal intervals but by due proportions. And although we have theſe ſucceſſive proportions for each ſemitone, it hath been found neceſſary in the conſtruction of organs, flutes, hautboys, baſſoons, &c. ſo to order the twelve intervals or ſemitones that their ſounds ſhall be in one and the ſame proportion thro' the octaves; and tho' each tone be rather bearing from its mathematical meaſure, yet the difference is ſo nice that very few ears can diſtinguiſh it: Therefore, we will follow the practical method of eſteeming all the half tones equal one with another.

N. B. Where I could find inſtances of inſtruction from other modern publications that were plain and eaſy, I have taken the liberty of inſerting them along with what other alterations and additions I thought was convenient for the learner.

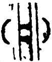
Note. I have omitted the gamuts for the ſeveral ſorts of inſtruments in general uſe, as numbers of each ſort abound; and have inſerted one general gamut of the notes which may ſerve for all.

The following is the Gamut of natural notes, whether vocal or inſtrumental.



The firſt thing to be obſerved is to learn the names of the notes upon the lines and ſpaces, by heart. Alſo due regard muſt be had to the cliffs ſet at the beginning of the five lines, which are an inlet to the knowledge of the notes; for if a note be placed on any part of the five lines, you cannot call it any thing till there is one or other of the cliffs ſet at the beginning: For which reaſon the lines of the gamut are divided into ſeparate fives, (or ſtaves,) expreſſing the different parts of muſic. For inſtance, the treble or uppermoſt five lines has G for its cliff, ſet at the beginning on the ſecond line of the ſtave from the bottom. The lower

line

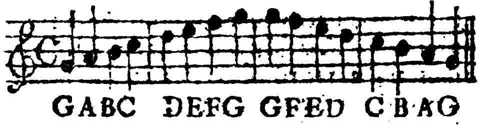
moſt part or baſs, has F for its cliff, and is commonly placed on the fourth line from the bottom. But there is another ſort of cliff that comes between the treble and the baſs, call'd the tenor cliff, thus (), and which may be ſet on any of the four loweſt lines; it being C on that line whereon it is put.

The gamut may be repeated as well aſcending as deſcending, and by different notes; but the diſtance from one note to the other muſt alſo be obſerved, and this only in aſcending. The certain intervals are as follows.

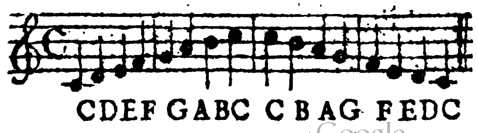
From G to A is a whole tone, from A to B is a whole tone, from B to C is half a tone, from C to D is a whole tone, from D to E is a whole tone, from E to F is half a tone, from F to G is a whole tone, and ſo on with never ſo many notes; which muſt aſcend in the ſame proportion of ſound as the firſt eight notes do; as every note bears the ſame union of ſound with its octave, whether it be an eighth above or below.

I ſhall inſtance an example or two of an octave: But for fear you ſhould not tune the notes exactly, you ought to get the aſſiſtance of a perſon ſkill'd in muſic, and let him ſing or play your eight notes with you, till you can do them without him.

Example.

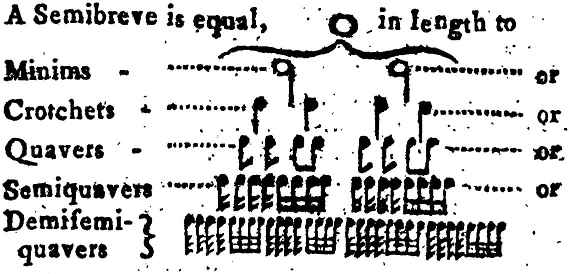


Example.

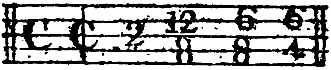


The next thing tow'rds the attainment of the knowledge of muſic, is to be well acquainted with the different meaſures of notes, which are under the denomination of time.

Of Time, or the length of notes.



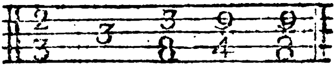
There are two ſorts of time, common and triple. Common time is known by ſome of the following marks or characters.



The firſt of theſe marks denotes the ſloweſt kind of movement, and contains a ſemibreve (or as many other notes as are equal to its length) in a bar, and muſt be held as long as you can diſtinctly tell 1, 2, 3, 4. The ſecond denotes a movement ſomewhat faſter than the former, and contains alſo a ſemibreve in a bar. The third denotes a brisk movement, and contains but one minim, or two crotchets, &c. in a bar. The fourth mark contains twelve quavers (or notes to their value) in a bar; the fifth ſix quavers in a bar, and the laſt ſix crotchets in a bar. The three laſt characters are fixed to jiggs, &c.

In a ſlow common time you muſt divide the bar into four equal parts, telling 1, 2, 3, 4, diſtinctly, putting your hand or foot down when you tell one, which muſt be at the beginning of the bar, and lifting it up when you tell 3, which muſt be in the bar. In a quick ſort of common time, you may divide your bar into two equal parts, only putting your hand or foot down at the firſt half of the bar and lifting it up at the ſecond half; but you muſt be exact in moving up and down. Triple time, whether quick or ſlow, muſt be divided in three equal parts, telling 1, 2, with your hand down, and 3 with it up: In this ſort of time you muſt keep your hand up but half the time you keep it down. Obſerve thro' the whole the down beat at the firſt note in every bar.

The following are marks of triple time.

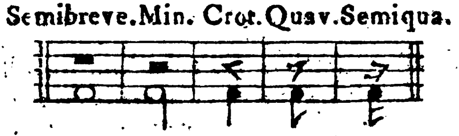


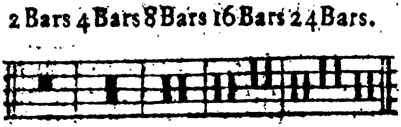
The firſt of theſe characters has three minims in a bar, and is the ſloweſt triple time in uſe. The ſecond contains three crotchets in a bar, and is fixed to minuets, and play'd quicker than the former. The third contains three quavers in a bar, and is the quickeſt. The fourth contains nine crotchets; the laſt nine quavers. Theſe laſt are rarely made uſe of, and then to jiggs.

Obſerve a diminutive note or grace is ſet before a real note, and is only meant to prepare that real note, and not reckoned into the time. Alſo when you find the figure 3 ſet over three ty'd notes, it ſignifies they are to be play'd and reckon'd in the time of two of thoſe notes.

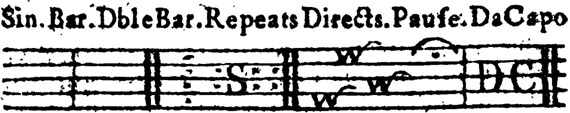
Note. A point or dot added to any note, whether minim, crotchet, &c. makes it half as long again; and muſt always be put on the right ſide of the note.

EXAMPLE of RESTS.





Note. A ſemibreve reſt is a whole bar in any time whatever.



A ſingle bar ſerves to divide the time according to its different meaſures, whether common or triple. A double bar ſerves to divide every ſtrain or part of a ſong or leſſon. A repeat ſignifies that ſuch a part of a ſong, &c. muſt be performed over again from the note over (or before) which it is put. A direct is put at the end of a ſtave, and ſerves to direct to the place of the firſt note in the next ſtave. A pauſe ſignifies that the note over which it is placed muſt be held out ſomewhat longer than the uſual time.—The ſame mark alſo denotes the end of a tune. Da Capo ſignifies the tune does not end there, but muſt be begun again, and play'd till you come to the mark 𝄐 aforemention'd to denote the end of the tune.

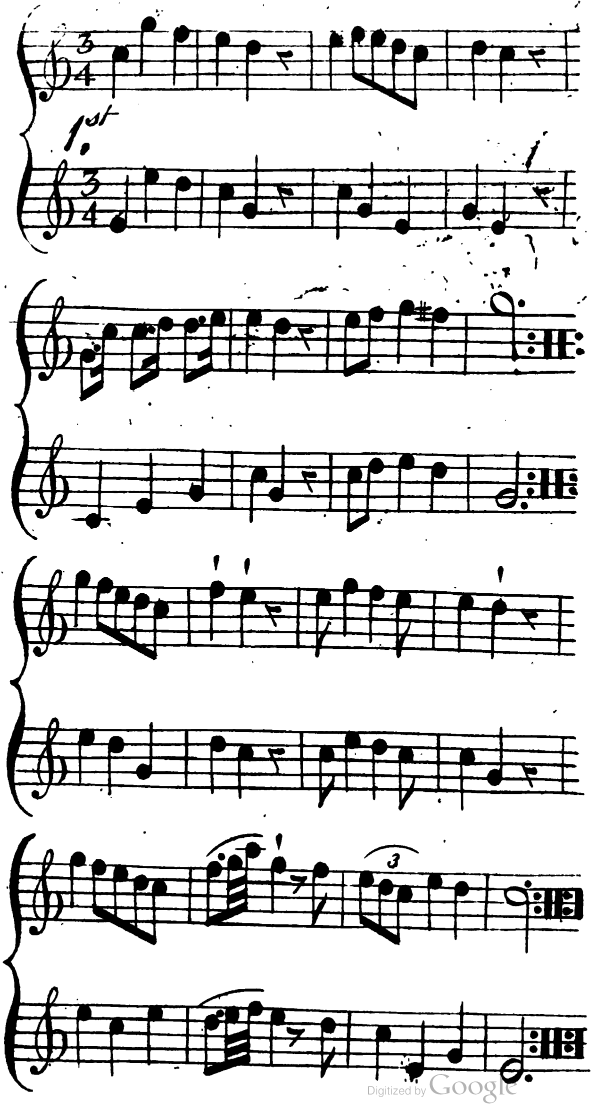
Of Flats and Sharps, &c.

Theſe characters are very ſignificant in muſic, and muſt be particularly regarded; viz. (♭) or Flat; (♯) or Sharp; (♮) or Natural. If a flat be placed before any note it denotes that ſuch note (and all the following in the ſame bar on the ſame line or ſpace except mark'd to the contrary) muſt be ſung or play'd half a note lower than its natural pitch. The ſharp is of a contrary nature; for whereas a flat takes away a ſemitone, or half a note, from the ſound of the note before which it is ſet, the ſharp adds a ſemitone to whatever note it is ſet before. For example, if a flat (or flats) be fixt at the beginning of any of the five lines, it not only affects every note on ſuch line or ſpace but alſo all the notes of that denomination thro' the whole movement. The ſame is likewiſe to be obſerved of the ſharps. A natural ſerves to reduce any note made flat or ſharp by the governing flats or ſharps placed at the beginning, to its primitive ſound, as it ſtands in the gamut.

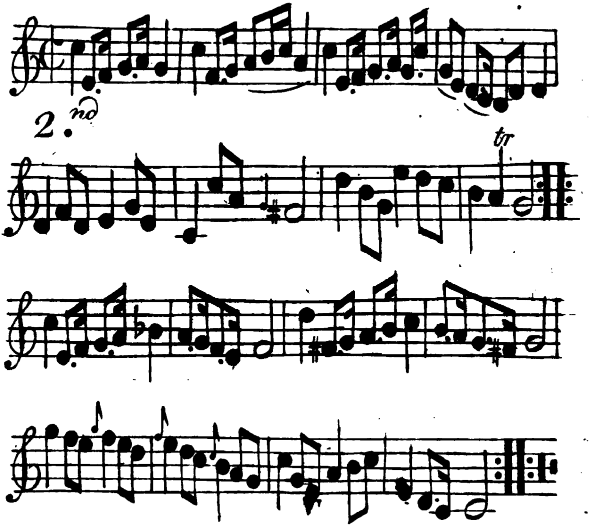
I ſhall next inſtance ſome ſelect pieces for leſſons, both for the voice and inſtrument: But you muſt be particular in obſerving the time, as beforemention'd; for no muſic can be agreeable to the performer unleſs he makes himſelf maſter of it; neither is it poſſible for ſeveral performers to agree exactly together without it.

N. B. The following eight pages of copperplates containeth ſix leſſons for different inſtruments, and three ſongs. The ſix leſſons are adapted to the under-mentioned inſtruments;

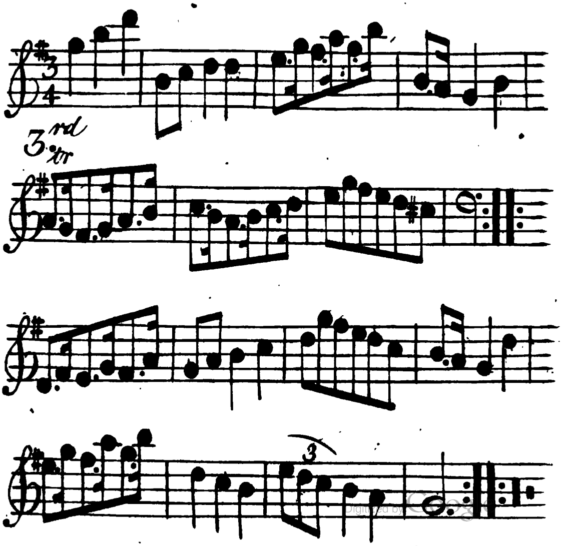
viz.
- 1ſt. For two French Horns.
- 2d. For the Guittar.
- 3d. For the German Flute.
- 4th. For ditto.
- 5th. For ditto.
- 6th. For the Violin.


1st
3

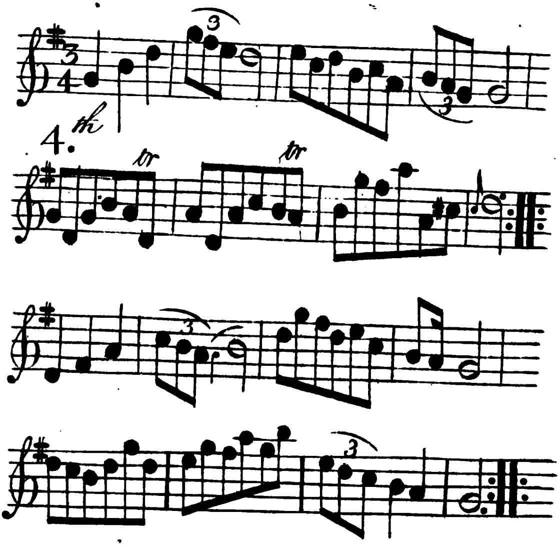
2.nd



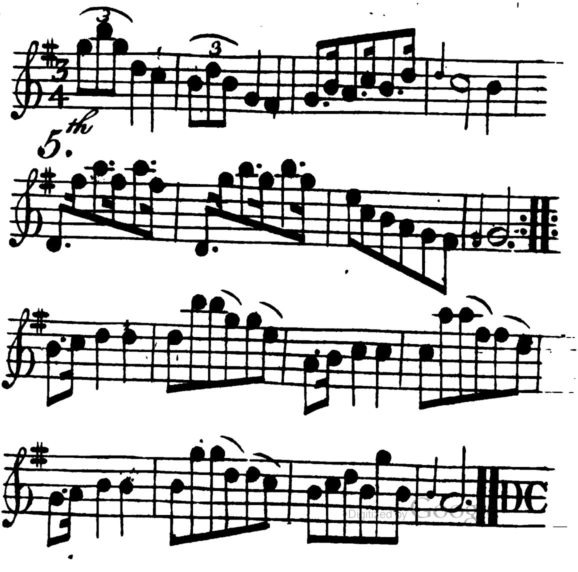
3.rd

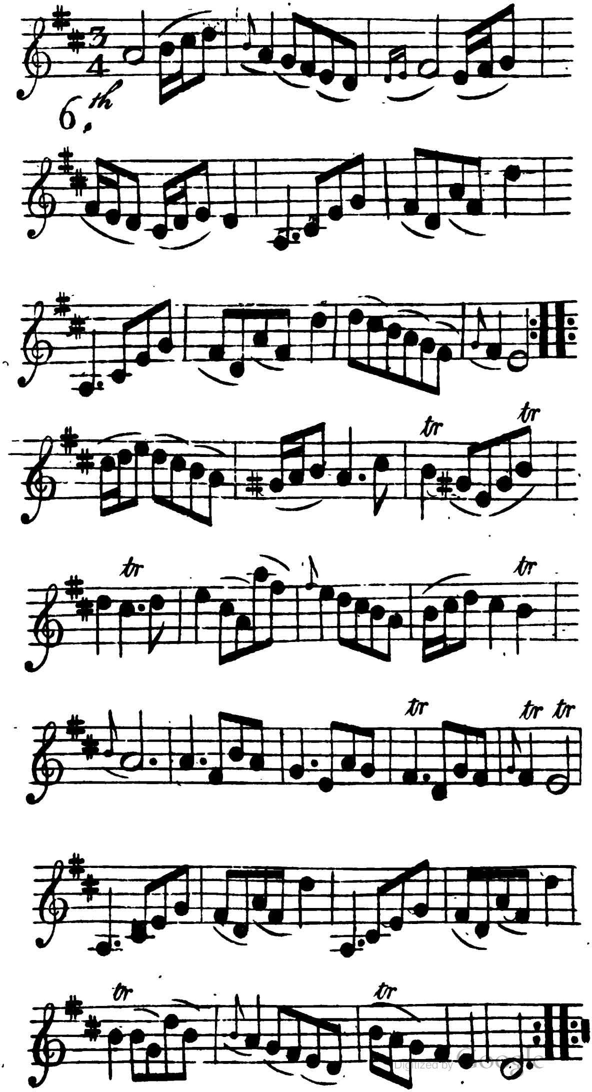


4.th

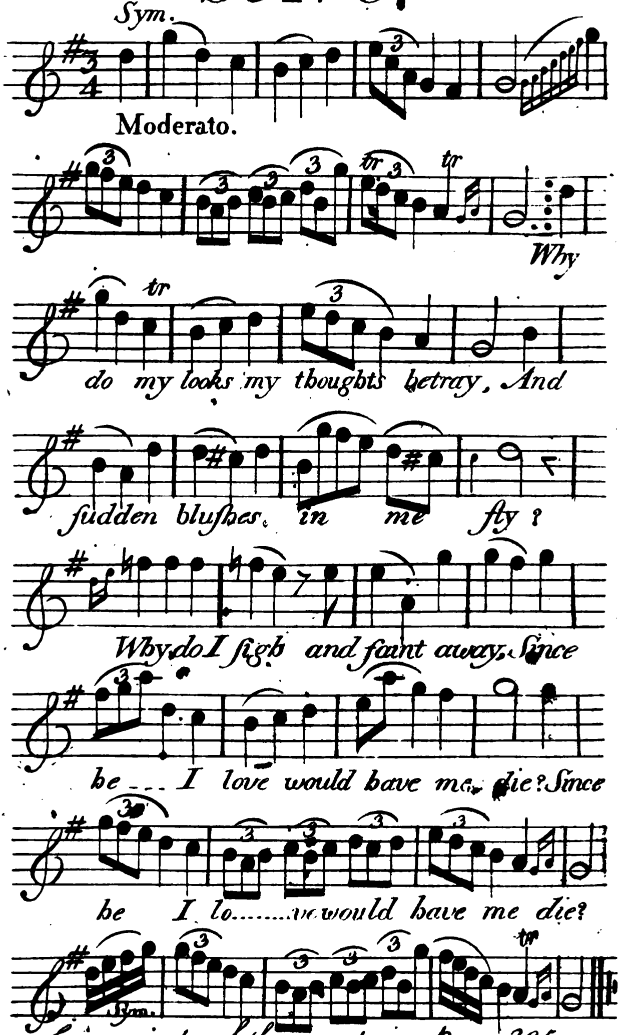


5.th



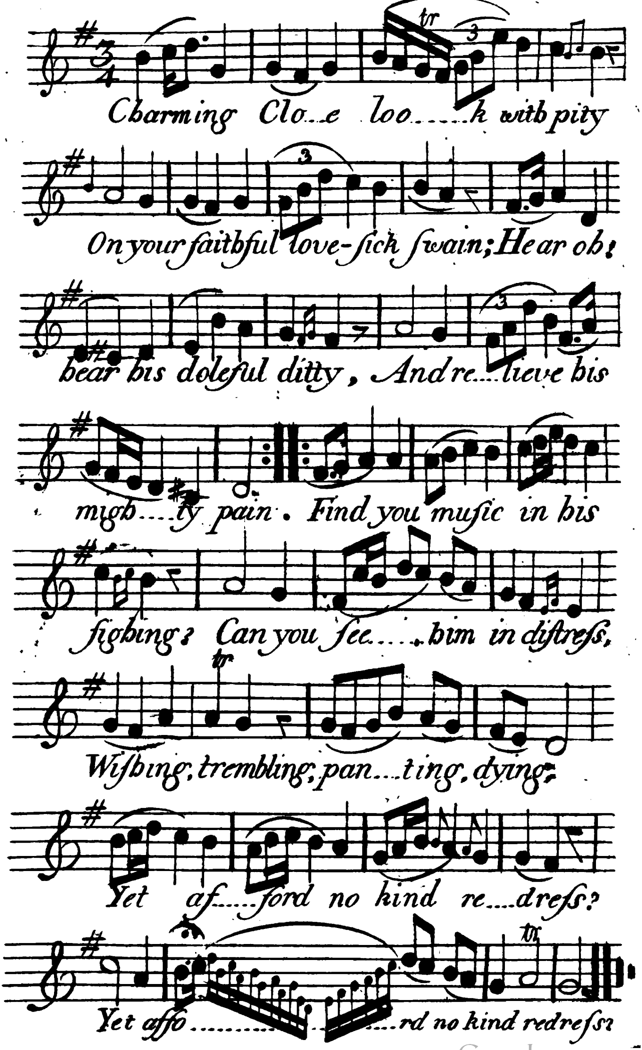
3
4
6th
tr
tr
tr
tr
tr
tr
tr
tr
tr
tr

SONG.

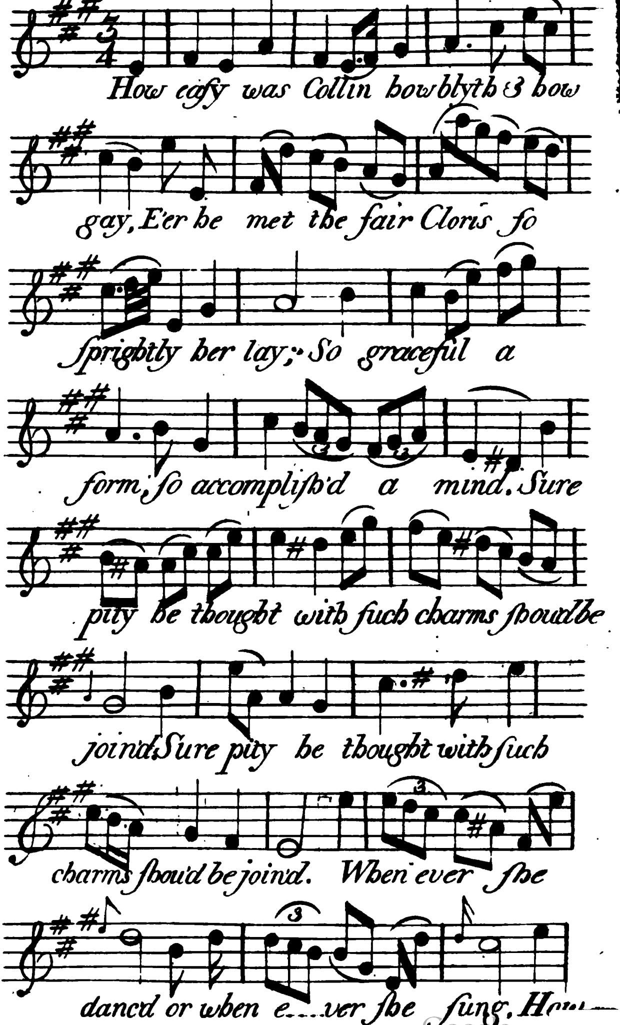

Sym.
Moderato.
Why
do my looks my thoughts betray, And
sudden blushes in me fly?
Why do I sigh and faint away, Since
he --- I love would have me die? Since
he I lo.......ve would have me die?
Sym.

See remainder of the words in page 305.

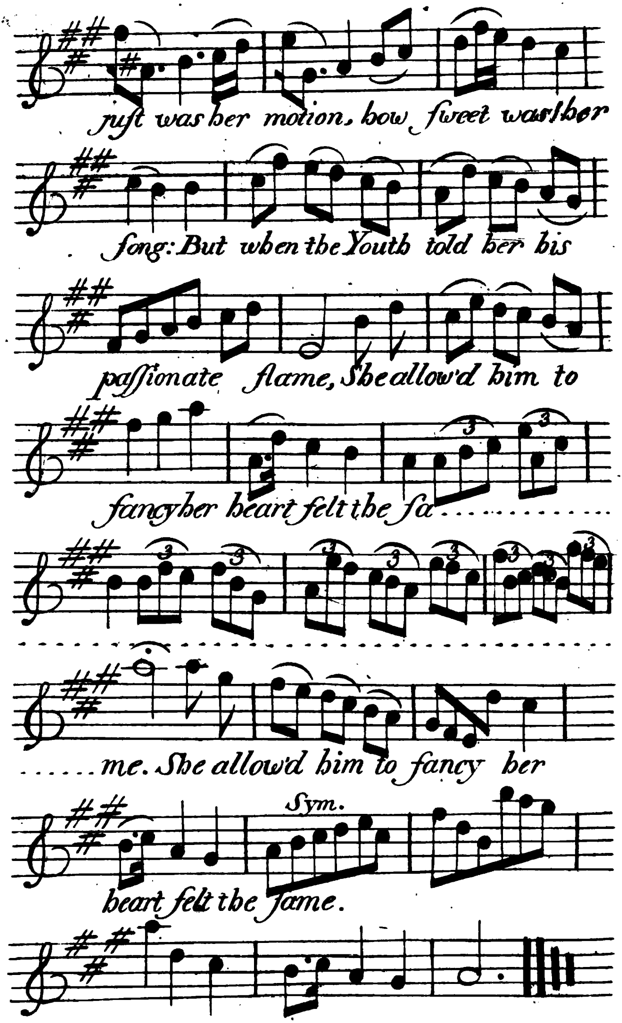
CHARMING CLOE.


tr
3
Charming Clo...e loo......k with pity
On your faithful love-ſick ſwain; Hear oh!
hear his doleful ditty, And re....lieve his
migh....ty pain. Find you muſic in his
ſighing? Can you ſee.....him in diſtreſs,
tr
Wiſhing, trembling, pan....ting, dying;
Yet af....ford no kind re....dreſs?
tr
Yet affo..........................rd no kind redreſs?

COLLIN and CLORIS.


How eaſy was Collin how blyth & how
gay, E'er he met the fair Cloris ſo
ſprightly her lay; So graceful a
form, ſo accompliſh'd a mind. Sure
pity he thought with ſuch charms ſhou'd be
join'd. Sure pity he thought with ſuch
charms ſhou'd be join'd. When ever ſhe
danc'd or when e-...ver ſhe ſung, How
3
3

Continued.


juſt was her motion, how ſweet was her
ſong: But when the Youth told her his
paſſionate flame, She allow'd him to
fancy her heart felt the ſa- - - - - - - - - - - -
- - - - - - me. She allow'd him to fancy her
Sym.
heart felt the ſame.

The remainder of the words to the firſt Song.

II.
Could I but once on him prevail,
To mingle with his joys my ſmart;
That he may feel what now I ail;
But I'm too young to ſhew ſuch art.

III.
Attractive Cupid be my care,
And look with pity on my pain:
Oh! break the chain that now I ware,
Or bind Ominton in the ſame.

IV.
Haſte to thy mother tell my grief,
To help an harmleſs injur'd maid;
That he may quickly ſend relief,
And ſave an heart that is betray'd.

Remainder of the words to Charming Cloe.

II.
Strephon, mov'd by lawful paſſion;
For no favours rudely ſues;
All his flame is out of faſhion,
Ancient honour for him wooes.
Love for love 's the ſwain's ambition;
But if that is deem'd too great,
Pity, pity his condition;
Say at leaſt, you do not hate.

III.
Should you, fonder of a rover,
Practis'd in the art of guile;
Slight ſo true and kind a lover,
Cloe, might not Strephon ſmile?

Yes: well pleas'd at thy undoing,
Vulgar lovers might upbraid ;
Strephon, conſcious of thy ruin,
Soon would be a ſilent ſhade.

Remainder of the words to Collin and Cloris.

II.

With ardour he preſt her to think him ſincere,
But alas! ſhe redoubl'd each hope and each fear;
She wou'd not deny nor ſhe wou'd not approve,
And ſhe neither refus'd him nor gave him her love.
Now cheer'd with compliance, now froze by diſdain,
He languiſh'd for freedom, but languiſh'd in vain ;
Till Thyrſis who pity'd ſo helpleſs a ſlave
Eas'd his heart of the pain by the council he gave.

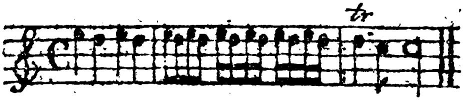
III.

Forſake her, ſaid he, and reject her awhile,
If ſhe loves you ſhe ſoon will return with a ſmile ;
You may judge of her paſſion by abſence alone,
And by abſence you'll conquer her heart or your own.
This advice he purſu'd, but the remedy prov'd
Too fatal alas! to the fair one he lov'd;
Which cur'd his own paſſion, and left her in vain
To ſigh for an heart ſhe cou'd never regain.

To

To make the Trillo or ſhake on the voice.

The trill or ſhake, (mark'd t, or tr.) is the chief grace in ſinging, and has a fine effect when well performed, To learn this you muſt raiſe your voice on one ſyllable, the diſtance of a whole tone, as in the following example. Firſt move your voice ſlowly, then faſter, by degrees, and it will come to you with a little practice; but you muſt be ſure to let the two notes be both heard diſtinctly.



The trill or ſhake is to be made on all deſcending prickt crotchets, and alſo when the note before it is in the ſame line or ſpace with it; likewiſe generally before a cloſe, either in the middle or at the end of a ſong.

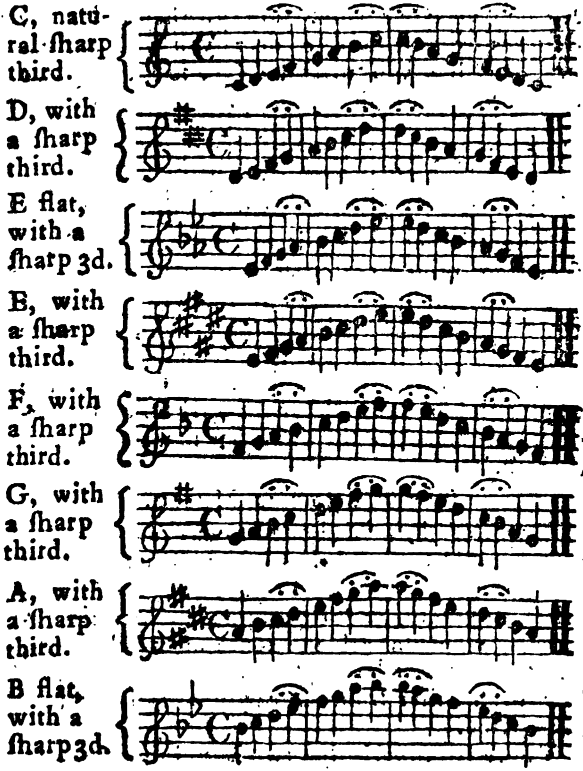
Note. If you wou'd practice ſinging, the application of the inſtrument will be of the utmoſt aſſiſtance to the attaining to a proficiency therein.

Of the KEYS in MUSIC.

The word key is adapted to one note choſen as principal to compoſe a piece of muſic in, it being the ending note, ſo called the fundamental or key-note, as it determines the certain intervals, wherein all the tones or whole notes, and ſemitones or half notes, which ought to follow each other from the key-note to its octave, take place.

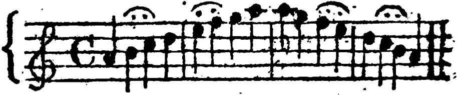
There is properly but two keys in muſic, one flat, the other ſharp; but by the help of flats and ſharps they have been varied to the number of twenty-four in the whole; one half of which are ſharp keys, and the other half flat keys: Thoſe moſtly in uſe are as follow.

A SCALE of SHARP KEYS.

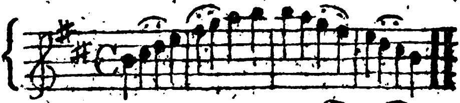


A SCALE of FLAT KEYS.

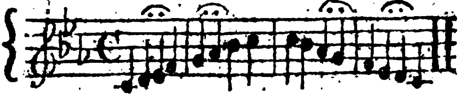
A, natural flat third.



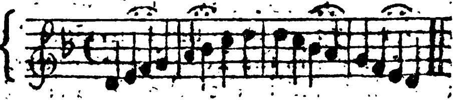
B, with a flat third,



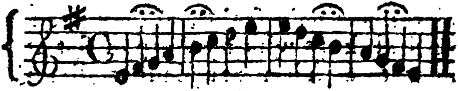
C, with a flat third.



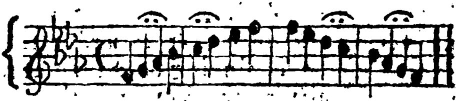
D, with a flat third.



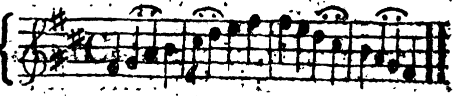
E, with a flat third.



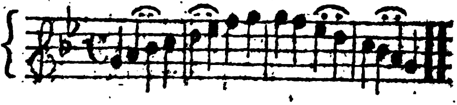
F, with a flat third.



F ſharp, with a flat 3d.



G, with a flat third.



Now for the better diſtinction of the whole tones and half tones in the keys, you muſt obſerve, that in the gamut, the half notes are naturally found betwixt E and F, and between B and C, whereas a *whole* tone is found betwixt all the other notes; and there being but ſeven different denominations, C, D, E, F, G, A, B, ſo there is but 7 different degrees of notes in the whole gamut (as before obſerv'd,) for all eights bear the ſame ſound as well as name: Likewiſe the flats and ſharps ſet at the beginning of a tune have the ſame effect on all their octaves above or below. As you may take whatever note you think proper for a key-note, provided you give a progreſſion to its octave, like unto that of C, if the key be ſharp; then ſharps or flats are placed after the cliff, in order to increaſe or leſſen, a ſemitone, thoſe intervals that might hinder that conformity: And as the half tones lie in particular places in the key, I have dotted the ſemitones included in the octave of every key in the afore-mention'd ſcales, as they are found in the natural keys with no ſharp or flat at the beginning, to which order and progreſſion all the other keys are modelled. Whence it will be found no very difficult matter to order the flats or ſharps at the beginning of a tune according to the conformity of the key, whether it be a flat key or a ſharp one.

For example, if we take D for a key-note, and would make its key agreeable to that of C, we muſt begin with the key-note aſcending; and as we reckon on, we ſhall find that both F and C muſt be ſharp'd, to make it conformable to the key of C.

The flat key of A may ſerve to illuſtrate all flat keys.

N. B. A key is known to be flat or ſharp not by what flats or ſharps are ſet at the beginning of a tune, but by the third above the laſt note, or key-note; for if a third contains two whole tones it is a ſharp key, but if only a tone and a half 'tis a flat key. Always name your key in reference to the baſs.

Of Tranſpoſition.

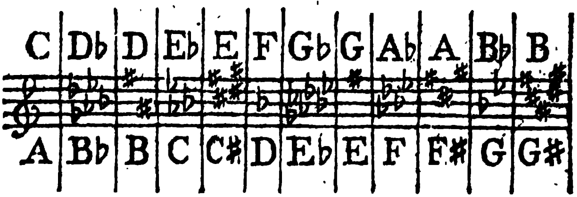
To tranſpoſe a ſong or leſſon that is too high, or too low, or in an improper key for a voice or inſtrument, you muſt firſt ſee what compaſs the tune requires; that is, how high and how low it goes, and accordingly take your meaſure. The laſt note of a tune, as before obſerved, tells you what key it is, whether A, B, C, D, &c. Therefore, ſuppoſe for example, you had a ſong or leſſon in the ſharp key of E, and you wanted it tranſpoſed into G, which is a third higher; look in the aforegoing ſcale for G with a ſharp third, where you'll find one ſharp, and that upon F; ſo placing the ſharp as at the beginning, and writing each note a third higher than it is in the copy, you have the tune right, in the deſired key.

You may tranſpoſe into any of the keys in the reſpective ſcale, obſerving the diſtance or number of notes from the key-note of your copy to the firſt note of the key you tranſpoſe into, and putting the ſame number of flats or ſharps at the beginning as in the ſcale.

Note. Whatever accidental flat, ſharp, or natural is met with from the beginning of a tune to the end, the ſame muſt be obſerved in that part of the tranſcript by ſome one or other of thoſe powers: Thus, when a natural is met with (this way) you muſt have regard to the effect it has on the note it ſtands before, whether it be of a flat or ſharp quality, and in ſuch caſe you muſt ſupply its place with either flat or ſharp, according to the effect it has in the copy. Obſerve the ſame order with the other powers; knowing a flat to one note is a ſharp to the note below it, except in a caſe where a natural half tone lies.

Now as there are ſeveral other places beſide what are mention'd in the aforeſaid ſcales which either of the keys may be tranſpoſed into upon occaſion; the reaſon aſcribed is, that every half note in the gamut ſhould have its fundamental or key-note.

The following is a SCALE *of all the* KEYS.



In this ſcale you'll obſerve ſtraight lines interſecting the common ſtave or five lines, and letters ſet both above and under the ſtave, ſignifying the keys to which the diviſions belong; each letter above the ſtave denotes a ſharp key anſwering to whatever flats or ſharps are under it, and the letter underneath denotes a flat key agreeing to the ſame flats or ſharps: As in the natural keys, C above the ſtave denotes a ſharp key, and A (a flat third below C) underneath the ſtave, ſignifies a flat key; and ſo with the reſt.

THE CONSTRUCTION OF ORGAN-PIPES, SPINETS, &c.

WHAT is to be obſerved in the conſtruction of organ-pipes is the hollow part or cavity from whence the ſound preceeds; therefore in this caſe we have nothing more to do than to find the dimenſions of ſuch particular cavities that will ſound any given note in the octave. And firſt the half of each particular dimenſion of any pipe gives the octave above, and the double of each particular dimenſion of any pipe gives the octave below.

Alſo we have the ſame reaſon for muſical ſtrings, &c. For inſtance, ſuppoſe two ſtrings of equal thickneſs, and ſtretched with equal weights, and have their lengths as 1 to 2, viz. one ſtring twice the length of the other, they will ſound an octave to each other; and conſequently in like manner the ſounds of any other ſtrings will be in proportion to their diameters and the weights that ſtretch them.

Suppoſe it was required to find a mean ſemitone for the conſtruction of organ-pipes, &c. ſo that their ſounds with regard to gravity or acuteneſs may be in the ſame proportion which altogether ſhall compleat the octave. In order to this, we know that the octave is known by this fraction $\frac{1}{2}$; and as there are 12 half notes in the octave, all melody and harmony are compoſed of theſe 12 notes; for the octave above or below are but the replicatives of the ſame ſounds in an higher or lower tone. Therefore the following table is the meaſure of the diſtance to any particular half tone required in the octave; and they are alſo factors to find the dimenſions of muſical chords and other inſtruments.

TABLE.

	1.00000
1	.94388
2	.89090
3	.84090
4	.79370
5	.74915
6	.70710
7	.66742
8	.62996
9	.59460
10	.56123
11	.52973
12	.50000

To apply the factors in the table to the conſtruction of organ-pipes.

Here you are to obſerve that the dimenſions of the cavity of ſome one pipe muſt be given; and each particular dimenſion muſt be multiplied by the factor in the table, and the product gives the like dimenſion of that pipe that will ſound the note belonging to that factor. Thus for example, ſuppoſe C the key-note, and the given dimenſions of the pipe to be 1.57 by 1.46, and 20.9 inches long. Now ſuppoſe I would have the dimenſions of that pipe to ſound G, viz. 7 half tones higher than the given pipe. Then 1.57 multiplied by .66742 gives 1.0478; 1.46 multiplied by .66742 gives .9744; and 20.9 multiplied by .66742 gives 13.9491; and thus you have the dimenſions of the pipe required.

The dimenſions for all the ſemitones in the above octave are as follow.

	Length	Breadth	Width
C	20.9000	1.5700	1.4600
C ſharp.	19.7271	1.4819	1.3701
D	18.6198	1.3987	1.3007
D ſharp.	17.5748	1.3202	1.2277
E	16.5883	1.2461	1.1588
F	15.6572	1.1762	1.0937
F ſharp.	14.7784	1.1101	1.0324
G	13.9491	1.0478	0.9744
G ſharp.	13.1662	0.9890	0.9197
A	12.4271	0.9335	0.8681
A ſharp.	11.7297	0.8811	0.8194
B	11.0713	0.8317	0.7734
C	10.4500	0.7850	0.7300

If you would have an octave below, multiply the dimenſions of any pipe by 2 and it gives the dimenſions of that pipe an octave below; and take ½ the dimenſions it gives an octave above: Proceed in this manner for double octaves either above or below.

In conſtructing round pipes of a cylindrical form, the ſame rules muſt be obſerved as with thoſe which are ſquared, and the given length and diameter only made uſe of in the operations.

To apply the factors in the table in conſtructing of muſical ſtrings.

In this caſe, multiply the length of the given ſtring by any of the factors in the table, and the reſult is the length of the ſtring that will ſound the note belonging the factor made uſe of if the thickneſs and tenſion are the ſame.

What muſt be the length of that ſtring to ſound 4 half tones above another that is 40 inches long?

.7937
40

Anſwer 31 7480 inches.

What muſt be the length of thoſe ſtrings to ſound the notes D, E, F, G, A, B, C, if the key-note C is 40 inches long?

.8909	multiplied by 40 produces	35.636 for D.
.7937		31.748 for E.
.74915		29.966 for F.
.66742		26.696 for G.
.5946		23.784 for A.
.52973		21.189 for B.
.5000		20.000 for C.

Suppoſe it is required to find the weight that any ſtring muſt be ſtretched with to ſound any particular note, by having given the weight requiſite to ſound the key-note.

Rule. Divide the given weight by the ſquare of the factor in the table, the quotient will give the weight required to ſound any note above the key-note, if the ſtrings are of equal lengths.

What muſt be the weight to ſtretch a ſtring 40 inches long that it may ſound an octave above the key-note, if the key-note is ſtretched with 7 pounds, and is 40 inches long alſo?

.5000

.5000

.25)7.00(28 pounds, the weight required.

50

200

200

From whence we find, that if the diameters and lengths of muſical ſtrings are equal, the times of the vibrations (and conſequently the note ſounded) will be inverſly as the ſquare roots of the weights which ſtretch them; for if the weights are as 1 to 4, whoſe ſquare roots are 1 and 2, then the times of vibration will be as 2 to 1, and conſequently they will ſound octaves to each other. Hence in conſtructing ſtringed inſtruments, as ſpinets, harpſichords, &c. a ſkilful artiſt will compound theſe proportions of the length, diameter, and tenſion of ſtrings, to very great advantage.

FINIS.

ERRATA.

Page. Line.
4 — 10 *for* underneatth *read* underneath.
6 — 22 *for* number *read* numbers.
16 — 24 *for* 4790×64 *read* 4790×84, &c.
29 — 9 *for* .92916 *read* .02916.
35 — 8 *for* factots *read* factors.
50 — 30 *for* gains of each *read* gains or loſſes of each.
110 — 3 *for* exttact *read* extract.
118 — 5 *for* of tower *read* of the tower.
138 — 31 *for* term *read* terms.
147 — 26 *for* what *read* what is.
175 — 17 *for* $\frac{1}{3}$ *read* $\frac{1}{8}$.
237 — 12 *for* meaſure *read* go.
242 — 31 *for* one one *read* one.
273 — 2 *for* may *read* many.
290 — 29 *for* ine *read* line.
ibid. — 30 *for* line *read* moſt.